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VOLTAGE REGULATION OF DIRECT CURRENT GENERATORS BY MEANS OF TRIODES

by N. A. J. VOORHOEVE and F. H. DE JONG. 621.313.13 : 621.385.3 : 621.316.722

The variations in the voltage from a direct current dynamo can be diminished by regulating the excitation current in relation to the terminal voltage. A very accurate regulation can be attained by using as excitation current the anode current of a triode amplifier valve whose input voltage is influenced by the variations of the terminal voltage.

A triode regulator working on this principle is described in this article. By means of various examples attention is drawn to the very effective characteristics of this arrangement.

A direct current generator delivers a voltage which exhibits certain variations depending on the load and on accidental circumstances. These variations can be diminished by regulating the excitation current in relation to the terminal voltage. This can be done by varying a resistance in the excitation circuit of the generator or of the separate exciter.

For rapid and accurate regulation it is desirable that the necessary variation of the resistance be effected by a device which functions as nearly as possible without time lag. Electronic tubes therefore offer an ideal means of carrying out this regulation.

In principle such regulation can be carried out with an arrangement such as that given in *fig. 1.* The circuit functions in the following way. When the terminal voltage v_b of the generator falls, due for instance to a variation in the load, the voltage at the grid of the triode T becomes less negative with respect to the cathode, so that the anode current increases. The anode current flows through the excitation circuit of the generator. The increase of the excitation current works against the decrease of the terminal voltage.

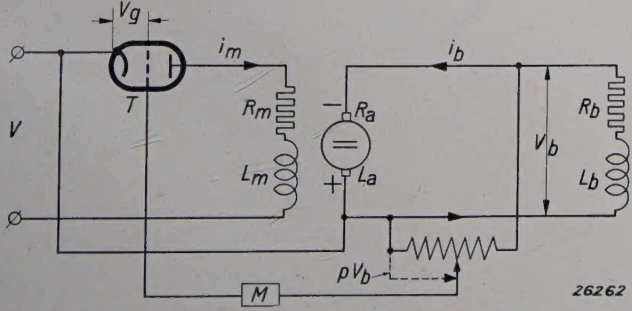

Fig. 1. Diagram of the circuit for regulation of the voltage with a direct current generator by a triode in series with the field winding.

For very accurate regulation it will in general be necessary not to conduct the variable terminals voltage directly to the grid of the regulating triode, but through a voltage amplifier. This voltage amplifier M is shown schematically in fig. 1.

Theory of the triode regulator

The most important requirements demanded of a voltage regulator are the following:

1. The regulation must be accurate. The ratio in which the voltage fluctuations are decreased by means of the regulator may be considered a measure of the accuracy.
2. The regulation must take place rapidly. Every regulation device has a certain lag, so that upon very sudden variations in load the voltage fluctuations are not diminished immediately.

If, for example the load on the generator with voltage regulation is suddenly removed, the voltage then varies as shown in *fig. 2.* It decreases suddenly and then increases gradually again to slightly more than its original value. According to the nature of the circumstances this adjustment to the final value may proceed exponentially or with a damped oscillation. In both cases we may characterize the speed with which the voltage approaches its final value by a time factor in a power of e, and this factor, the damping constant, forms a measure of the speed of regulation.

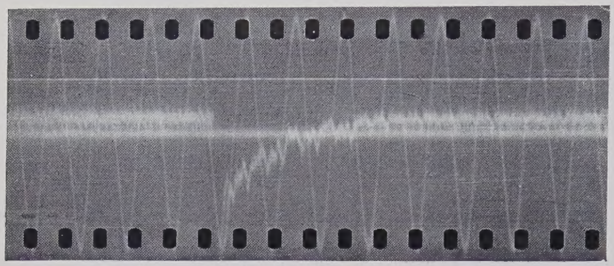

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Fig. 2. Variation of the terminals voltage upon sudden application of the load.

In order to examine the possibilities offered by the triode regulator in these two respects, we shall study the action of the apparatus mathematically. The following symbols will be used (see fig. 1):

L_a, R_a	self-inductance and resistance of the armature circuit of the generator,
L_b, R_b	self-inductance and resistance of the load applied,
i_b, v_b	current and voltage supplied by the generator,
pv_b	the portion of v_b conducted to the regulator,
m	amplification factor of voltage amplifier M,
i_m, v_m	excitation current and voltage,
L_m, R_m	self-inductance and resistance of the excitation circuit,
μ, v_g, R_i	amplification factor, grid voltage and internal resistance of the regulator triode,
v_{g0}, i_{m0}, i_{b0}	values of v_g, i_m, i_b corresponding to an arbitrarily chosen stationary state.

The following equations can be set up. For the armature circuit:

$$(L_b + L_a)\frac{\mathrm{d}i_b}{\mathrm{d}t} + (R_b + R_a)\,i_b = \mathrm{EMF} \quad \cdot\ \cdot \quad (1)$$

With slight variations of the excitation current the EMF will vary linearly with i_m:

$$\mathrm{EMF} = v_0 + c_1\,i_m \quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \quad (2)$$

The current i_m comes from the triode circuit. In this circuit:

$$(i_m - i_{m0}) = \frac{\mu}{R_i}(v_g - v_{g0}) + \frac{1}{R_i}(v_m - v_{m0}),$$

where:

$$v_m - v_{m0} = -R_m\,(i_m - i_{m0}) - L_m\frac{\mathrm{d}i_m}{\mathrm{d}t}.$$

Thus:

$$\mu\,(v_g - v_{g0}) = (R_i + R_m)\,(i_m - i_{m0}) + L_m\frac{\mathrm{d}i_m}{\mathrm{d}t}\cdot (3)$$

The voltage fluctuations in v_g originate from fluctuations in the voltage of the generator by voltage division and amplification in M.

$$v_g - v_{g0} = -mp\cdot(v_b - v_{b0}) =$$

$$= -mp\left[c_1\,(i_m - i_{m0}) - R_a\,(i_b - i_{b0}) - L_a\frac{\mathrm{d}i_b}{\mathrm{d}t}\right]\cdot \quad (4)$$

If we eliminate $v_g - v_{g0}$, i_m and $\dfrac{\mathrm{d}i_m}{\mathrm{d}t}$ from equations (1) to (4) we find for i_b a homogeneous linear differential equation of the second order:

$$P\frac{\mathrm{d}^2 i_b}{\mathrm{d}t^2} + Q\frac{\mathrm{d}i_b}{\mathrm{d}t} + R\left(i_b - \frac{S}{R}\right) = 0, \quad \cdot\ \cdot\ \cdot\ \cdot \quad (5)$$

in which:

$$P = L_m\,(L_b + L_a),$$

$$Q = L_b\,(R_i + R_m + mp\mu c_1) + L_a\,(R_i + R_m) + {}$$
$$+ L_m\,(R_b + R_a),$$

$$R = R_b\,(R_i + R_m + mp\mu c_1) + R_a\,(R_i + R_m),$$

$$S = (v_0 + c_1\,i_{m0})\,(R_i + R_m) + {}$$
$$+ mp\mu\,c_1\,(v_0 + c_1\,i_{m0} - R_a\,i_{b0})$$
$$= (v_0 + c_1\,i_{m0})\,(R_i + R_m) + mp\mu\,c_1\,v_{b0}\,.$$

As is known, the general solution of differential equation (5) can be written in the form of the sum of two powers of e:

$$i_b - S/R = A_1\,e^{-a_1 t} + A_2\,e^{-a_2 t}. \quad \cdot\ \cdot \quad (6)$$

If this expression is introduced into equation (5) the following quadratic equation is obtained for a_1 and a_2:

$$P\,a^2 - Q\,a + R = 0\,.$$

The solutions are:

$$a_1,\,a_2 = \frac{Q}{2P} \pm \sqrt{\frac{Q^2}{4P^2} - \frac{R}{P}}, \quad \cdot\ \cdot\ \cdot \quad (7)$$

We shall see later from examples that the square root in equation (7) can be real as well as imaginary. In the first case the current will approach its final value exponentially according to the formula below; in the second case in the form of a damped oscillation. The following formulae are valid for the two cases respectively:

$$i_b = \frac{S}{R} + A_1\,e^{-\left(\frac{Q}{2P} + \sqrt{\frac{Q^2}{4P^2} - \frac{R}{P}}\right)t} + A_2\,e^{-\left(\frac{Q}{2P} - \sqrt{\frac{Q^2}{4P^2} - \frac{R}{P}}\right)t}$$

(8a)

and:

$$i_b = \frac{S}{R} + e^{-\frac{Q\,t}{2P}} \left(A_1' \cos t \sqrt{\frac{R}{P} - \frac{Q^2}{4P^2}} + \right.$$

$$\left. + A_2' \sin t \sqrt{\frac{R}{P} - \frac{Q^2}{4P^2}} \right), \quad \cdot \quad (8b)$$

where A_1' and A_2' as well as A_1 and A_2 are arbitrary constants. P, Q, R and S are positive according to definition. From this it follows that the exponents of the powers of e of equation (8) are always negative; instability is thus out of the question.

The speed of regulation is determined by different quantities depending on whether we are concerned with equation (8a) or (8b).

The first case, exponential adjustment, occurs when $Q^2 > 4PR$. In this case the speed of regulation is determined by the smaller of the two exponents of the powers of e of equation (8a). This is:

$$a_2 = \frac{Q}{2P} \left(1 - \sqrt{1 - \frac{4PR}{Q^2}} \right).$$

In some practical cases Q^2 will be much greater than $4\,PR$, so that we may approximate the term $\sqrt{1 - 4\,PR/Q^2}$ by $1 - 2\,PR/Q^2$. We then obtain:

$$a_2 = \frac{R}{Q}.$$

The second case, $Q^2 < 4\,PR$, gives a damped oscillation according to equation (8b), the damping constant (β) is:

$$\beta = \frac{Q}{2P}.$$

Examples in the following will show that a_2 as well as β may easily be equal to 3000 sec^{-1}, for instance, and the regulation then takes only $^1/_{3000}$ sec.

Further discussion of the result

In the case of an oscillatory adjustment the following value is found for the damping constant $\beta = \frac{Q}{2\,P}$:

$$\beta = \frac{L_b\, m p \mu\, c_1}{2\, L_m\, (L_b + L_a)} + \frac{1}{2}\,\frac{R_i + R_m}{L_m} + \frac{1}{2}\,\frac{R_b + R_a}{L_b + L_a} \quad \cdot \cdot \quad (9)$$

and for the frequency of oscillation: $\sqrt{\frac{R}{P} - \frac{Q^2}{4\,P^2}} = \omega$:

$$\omega^2 = \frac{R_b\,(R_i + R_m + m p \mu\, c_1) + R_a\,(R_i + R_m)}{L_m\,(L_b + L_a)} - \beta^2 \cdot (10)$$

In the case of a dead beat adjustment ω^2 is negative; the damping is then determined by:

$$a_2 = \beta - \sqrt{-\omega^2}.$$

In general the loading resistance has an inductive component ($L_b > 0$); in this case β and ω^2 as functions of the amplification factor m vary qualitatively as shown in *fig. 3a*. For a sufficiently high amplification ω^2 will become negative; the damping constant, which at first increases with m, then becomes smaller again and approaches the final value R_b/L_b, which no longer depends upon the generator, but only on the loading impedance. If, however, the load consists of a purely ohmic resistance, then β and ω^2 vary according to fig. 3b; an oscillatory adjustment cannot now be made exponential by increasing the amplification, and the damping of the oscillation is not affected in this case.

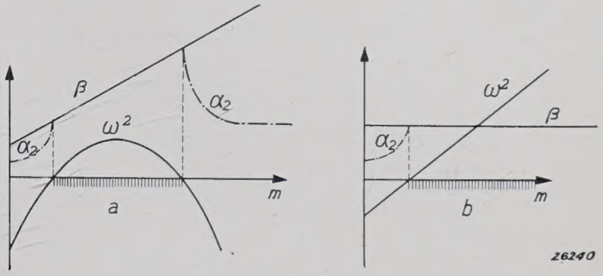

Fig. 3. Variation of a_2, β and ω^2 as functions of the amplification m. In the cross hatched area of the abscissa an oscillating adjustment must be made; outside of the area the adjustment is exponential.
a) The loading impedance has self-inductance.
b) The load is a purely ohmic resistance.

In order to estimate the accuracy of the regulation we must find out how the final value of

$$v_b = i_b\, R_b, \text{viz. } v_b = \frac{S}{R}\, R_b\,,$$

depends upon the loading resistance R_b (the self-inductance L_b has no influence on the final value). If R_{b0} and R_b are two different values of the loading resistance, then the difference between the corresponding generator voltages is:

$$v_b - v_{b0} = - \frac{(v_0 + c_1\, i_{m0})\, \dfrac{R_a\,(R_b - R_{b0})}{R_b R_{b0}\,(1 + R_a/R_{b0})}}{\dfrac{m p \mu\, c_1}{R_i + R_m} + 1 + \dfrac{R_a}{R_b}} \quad (11)$$

If voltage regulation is not applied the same expression is obtained with $m = 0$:

$$(v_b - v_{b0})_z = - \frac{(v_0 + c_1\, i_{m0})\, \dfrac{R_a\,(R_b - R_{b0})}{R_b R_{b0}\,(1 + R_a/R_{b0})}}{1 + \dfrac{R_a}{R_b}} \quad (12)$$

The accuracy of regulation was defined above by

$$N = \frac{(v_b - v_{b0})_z}{(v_b - v_{b0})}\,,$$

and from (11) and (12), therefore:

$$N = \frac{\dfrac{mp\mu\, c_1}{R_i + R_m} + 1 + \dfrac{R_a}{R_b}}{1 + \dfrac{R_a}{R_b}} .$$

In most practically occurring cases $mp\mu c_1 \gg R_i + R_m$ and $R_a \ll R_b$, so that by close approximation:

$$N = \frac{mp\mu\, c_1}{R_i + R_m} . \quad \dots\dots \quad (13)$$

It follows from equation (13) that the accuracy of regulation increases proportionally with the voltage amplification.

Example:

In the case of a direct current dynamo of 460 volts, 2.5 kW with a triode regulator the following values were observed:

$m = 360$ $\qquad R_a = 7.2$ ohm

$p = 1$ resp. 0.1 $\qquad R_b = 585$ ohm

$c_1 = 1700$ ohm $\qquad R_i = 360$ ohm

$L_a = L_b = 0.155$ H $\qquad R_m = 1675$ ohm

$L_m = 82.5$ H

For $p = 0.1$ a damped oscillation and for $p = 1$ a dead beat adjustment is found. With the help of the values given above the following values can be calculated for $p = 0.1$:

$$P = 82.5\,(0.155 + 0.155) = 25.6$$

$$Q = 0.155\,(360 + 1675 + 0.1 \cdot 360 \cdot 6 \cdot 1700) + 0.155\,(360 + 1675) + 82.5\,(585 + 7.2) = 106500$$

$$R = 585\,(360 + 1675 + 0.1 \cdot 360 \cdot 6 \cdot 1700) + 2\,(360 + 1675) \cong 216 \cdot 10^6$$

and for $p = 1$:

$$P = 25.6 \quad , \quad Q = 620\,000 \quad , \quad R \cong 216 \cdot 10^7 .$$

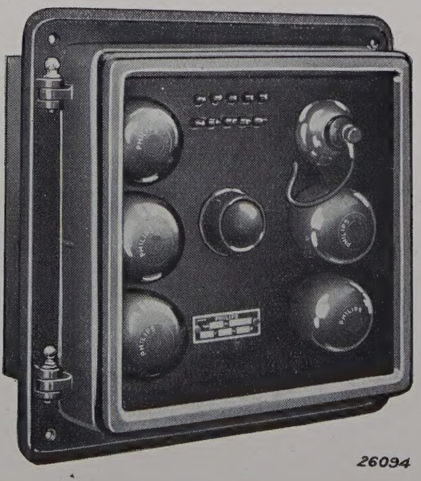

Fig. 4. Philips voltage regulator type 4142.

From this it follows that:

for $p = 0.1$: $\beta = 2080$ sec^{-1}; $N = 180$,

for $p = 1$: $\alpha_2 = 4200$ sec^{-1}; $N = 1800$.

The lag is therefore only several ten thousandths of a second, and the voltage fluctuations in the last case are reduced to practically $^1/_{2000}$. This shows that a very rapid and very accurate regulation may be obtained by the use of triodes.

Practical construction of the triode regulator

The Philips voltage regulator, type 4142, is reproduced in *fig. 4*, the diagram of the circuit in *fig. 5*. The method of functioning agrees in the main with the principle illustrated in fig. 1 and it can easily be examined by means of the diagram given in fig. 5.

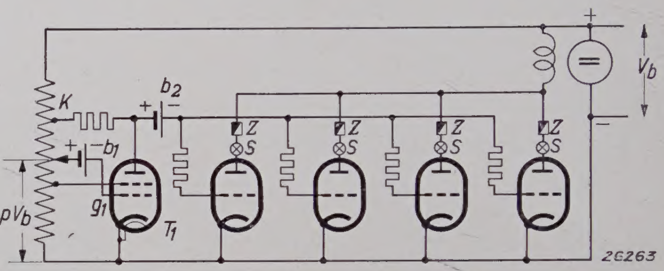

Fig. 5. Diagram of a voltage regulator with triodes.

The voltage amplifier M is a resistance amplifier. The tetrode T_1 serves as amplifier valve. The regulation voltage pv_b is taken from the potentiometer P and conducted *via* the grid battery b_1 to the control grid g_1 of the tetrode. When the terminal voltage v_b increases the anode current of the tetrode also increases, and the anode voltage decreases in consequence. This variation in voltage is transmitted *via* a second grid battery b_2 to the grid of a number of triodes in parallel, which supply the excitation current. The anode voltage of the triodes, like that of the tetrode, is supplied by the generator. In order, however, if necessary to use the tetrode with a higher anode voltage, point K (anode voltage tetrode) has an extra external lead.

The following points may be noted in regard to the arrangement:

1. The regulator works with several valves connected in parallel. This gives increased security during action. If one of the valves develops a defect the regulation proceeds practically unchanged with the remaining valves.
2. In the anode circuit of each regulator triode there is a small pilot lamp s and a fuse z. The pilot lamp is extinguished as soon as the anode current of the triode becomes abnormally small

due to a defect. The fuse burns through, when the current becomes abnormally great due to a short circuit.

3. The grids of the regulator triodes are connected to the grid battery through high resistances. This prevents the total anode current (excitation current) from becoming too great in case of a short circuit between grid and cathode.

Some practical applications of the triode rapid regulator

The special characteristics of the triode rapid regulator, namely, very high accuracy of regulation and very rapid regulation, make it suitable for use where a very constant voltage is essential.

It may be used in laboratories, schools, institutions for the calibration of instruments and in towing tanks for ship design. We shall briefly describe the last two applications, where an expensive accumulator battery needing much upkeep can be replaced by a much less expensive converter installation with a triode regulator needing little attention.

Use of the triode regulator in a calibrating station

For the calibration of electrical meters the following is the usual method.

Separate alternating current generators driven by a common direct current motor are used to feed the voltage and current coils. In order to be able to measure at different phase displacements the voltage generator is provided with a stator which may be turned about its axis.

During a calibration process the voltages and the frequency of both generators must remain strictly constant; this requirement is met when the direct current motor is connected with a source of voltage which has practically a constant voltage. A battery of accumulators is commonly used as source of voltage; the triode regulator makes it possible to replace the battery by a converter with direct current dynamo whose terminals voltage is kept strictly constant by the regulator.

A triode regulator for such an installation is shown in *figs. 6* and *7*.

The converter consists of a three-phase synchronous motor coupled with two direct current generators of 22 kW, which feed the three wire system of 2 × 125 volts for the calibrating units.

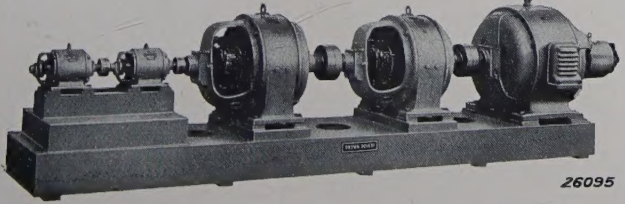
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Fig. 6. Converter consisting of a three-phase synchronous motor and two direct current generators of 22 kW.

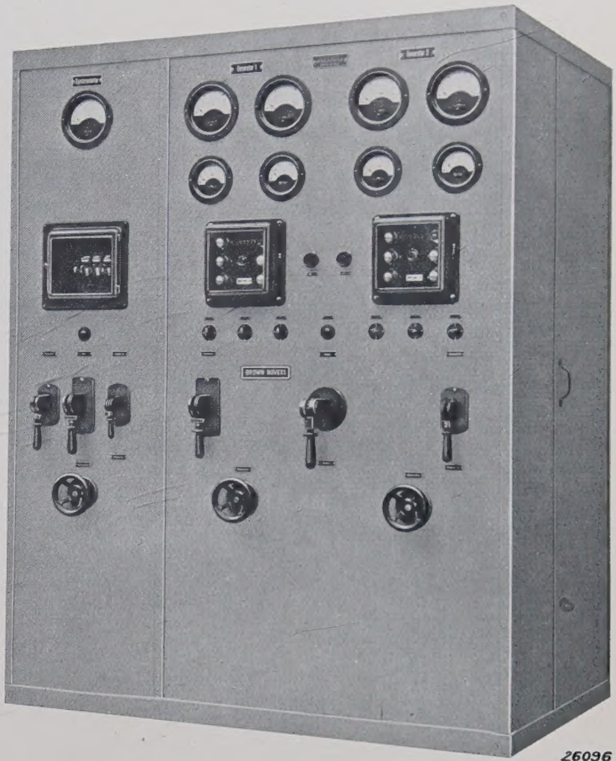
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Fig. 7. Switch board for converter of fig. 6 with two built-in Philips voltage regulators type 4142.

Fig. 6 shows the converter, fig. 7 the switch box with Philips triode rapid regulators type 4142 built in.

The values $R_m = 1055$ ohms and $c_1 = 330$ ohms are obtained by measurement from the generator.

The triode regulator is so adjusted that $p = 1$, $m = 500$; the regulator triodes have an amplification factor $\mu = 6$ and an internal resistance of 1500 ohms. There are five triodes connected in parallel, so $R_i = 300$ ohms. From this it follows that:

$$N \cong \frac{6 \cdot 500 \cdot 330}{1055 + 300} \cong 730.$$

The fall in voltage from no load to full load without regulation is 6 volts, with regulation the fall in voltage is decreased to $6/730 \cong 0.008$ volts.

The terminal voltage is 125 volts; the deviation from the average terminals voltage with regulation is therefore only ± 0.03 per cent. It has been proved by measurements that in practice, upon sudden variations in the load, this very high accuracy of regulation is actually attained. After longer times the terminal voltage will in general show somewhat greater variations, because the grid battery voltages also exert an influence on the terminal voltage. The percentage change of v_b is equal to the percentage change of the battery voltage. With good quality batteries these changes proceed very slowly, so that it is practically possible to keep the terminals voltage constant within 0.1 per cent.

Application in the case of a towing tank

A triode regulator for the converter installation of a towing tank was supplied to the Netherlands Shipbuilding Testing Station in Wageningen. The towing truck used to draw models of ships through the tank runs on four wheels. Each wheel is driven by a direct current motor of about 20 H.P. The motors are excited separately from direct current mains of 220 volts. The armatures of the motors are connected in series with a maximum armature voltage of 145 volts per motor. The running speed of the motors is regulated by varying the armature voltage, and for this purpose the armatures, in series with each other, are fed from a circuit with a variable voltage from 8 to 600 volts.

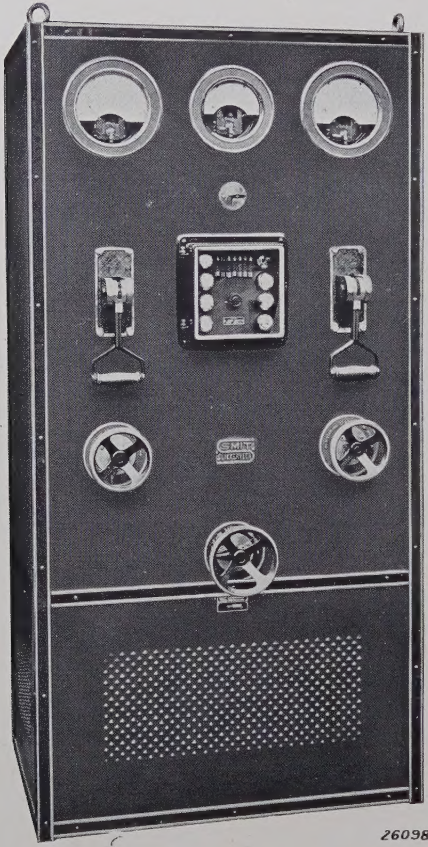
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Fig. 8. Switch board of converter for 220 volts direct current with built-in rapid regulator.

Since the speed of the towing car during a test must be strictly constant, it is necessary that the voltages of the two direct current circuits be kept accurately constant. This is done in the following way.

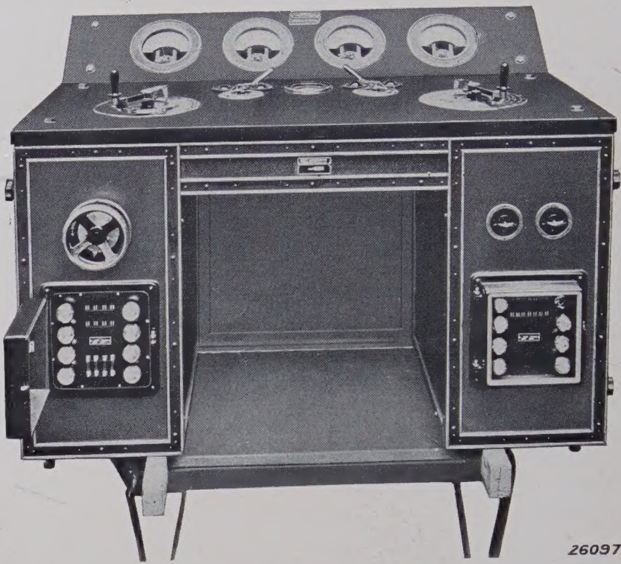
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Fig. 9. Switch table for 72 kW Ward-Leonard converter with built-in rapid regulators.

The circuit of 220 volts is fed by a converter with a synchronous three-phase motor and a compound dynamo whose terminal voltage is regulated by a triode regulator. For feeding the variable 600 volts circuit, which supplies the energy to the motors of the car, a battery of accumulators is usually employed. In order to avoid this expensive and elaborate installation in the testing station,

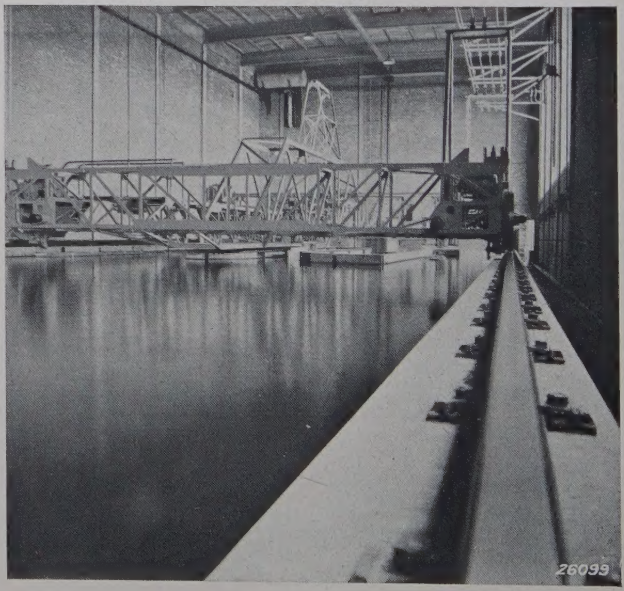
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Fig. 10. Truck for towing models of ships at the testing station in Wageningen. The truck runs on two rails over a tank 10.5 m wide, 5.5 m deep and 160 m long. It is driven by four motors each of 20 H.P. fed with a voltage kept constant by means of a voltage regulator.

the battery is replaced by a Ward-Leonard converter, consisting of a direct current dynamo of 72 kW with a 1.5 kW exciting dynamo supplied from the 220 volts mains, both driven by a synchronous three-phase motor.

The terminals voltage of the direct current dynamo is kept strictly constant — at all values used between 8 and 600 volts — by a triode rapid regulator which is included in the field circuit of the exciting dynamo.

Fig. 8 show the switch board belonging to the 220 volts converter with Philips triode regulator; *fig. 9* shows the control table for the Ward-Leonard converter; the triode rapid regulator may be seen in front. Since the number of regulator triodes had to be doubled in this case, they are housed in two similar cabinets. In *fig. 10* may be seen the towing truck which runs on two rails over a tank 10.5 m wide and 160 m long.

The installation has already been working for several years. The saving obtained by the installation of converters with triode rapid regulators instead of batteries of accumulators has in this case amounted to £ 1650.—.

THE BEHAVIOUR OF AMPLIFIER VALVES AT VERY HIGH FREQUENCIES

M. J. O. STRUTT and A. VAN DER ZIEL. 537.545 : 621.3.029.6 : 621.936.645.31

The properties of amplifier valves may be characterized by four admittances (reciprocals of impedances): the slope, the output admittance, the input admittance and the reaction admittance.

These admittances are in general complex. The real part is constant at broadcasting frequencies, and can therefore be considered as a resistance; the imaginary part is proportional to the frequency and may be considered as a condenser.

With ultra short waves there are deviations from this behaviour. These deviations may be ascribed, first, to the transition times of the electrons between the electrodes of the valve, and second, to self-inductances and mutual inductances between different conductors.

In this article the behaviour is studied of the four admittances in the case of ultra short waves, and the causes of the behaviour observed in each of the four cases are discussed individually.

Introduction

The properties of an amplifier valve are in general dependent on the frequency of the alternating voltage which is amplified by the valve. For some of the quantities the dependence on frequency is obvious. This is so, for example, in the case of the capacitive conductivity between different electrodes of the valve. Other quantities, such as the slope, are, in the region of broadcasting wave lengths (> 200 m), practically independent of the frequency and exhibit deviations from this behaviour only at very high frequencies. In the latter case less obvious phenomena play a part, and it is a discussion of these phenomena and their consequences which forms the main subject of the article.

The action of a radio valve may be described for sufficiently small alternating voltages between cathode and grid, and between cathode and anode, by the following equations respectively:

$$\left.\begin{aligned} i_a &= A\,v_g + B\,v_a \\ i_g &= C\,v_g + D\,v_a \end{aligned}\right\} \quad . \; . \; . \; . \quad (1)$$

The first equation expresses the fact that the current i_a at the output side (anode) of the valve has a linear relation to the alternating voltage v_g at the input side and v_a at the output side. The second equation gives an analogous relation for the current at the input side. The quantities A, B, C, D are determined by the various valve constants and have in general complex values.

We see from the equation that the quantities A to D indicate the ratio of a current to a voltage.

They are therefore reciprocals of impedances, in other words, admittances. We shall indicate them further as characteristic admittances of an amplifier valve.

The significance of the characteristic admittances may be understood by first short circuiting the output side of the valve and then repeating the process with the input side. In the first case $v_a = 0$ and

$$i_a = A\,v_g; \quad i_g = C\,v_g.$$

C is called the input admittance because it gives the relation between the input current and the input voltage, and A is the slope (transconductance) of the valve.

In the second case $v_g = 0$ and

$$i_a = B\, v_a; \quad i_g = D\, v_a.$$

B is called the output admittance and D the reaction admittance.

For a systematic study of the influence of the frequency on the properties of an amplifier valve, it is best to find out how each of the four admittances depends upon the frequency. Before doing this, however, we shall first call attention to several general considerations.

Cold and hot admittances of an amplifier valve

The input terminals of an amplifier valve consist of cathode and control grid, the output terminals of cathode and anode. When the cathode is not heated, there is no direct connection between the terminals in either case, at least when we may disregard insulation leakages or subtract their influence on the admittances. When in this article we speak of a definite admittance we shall always mean *the value which remains after the insulation leakages and the dielectric losses measured on the cold valve have been subtracted.* In the case of a cold valve the admittances will in the first place be made up of capacitive conductivities between the electrodes. In the second place inductive and ohmic resistances of the leads may play a part. These impedances are in series with the capacitive impedances between the electrodes, and are comparatively much smaller at broadcasting frequencies, so that we may neglect their influence as a approximation. The substitution diagram of the cold valve [1]) is then represented by *fig. 1a.*

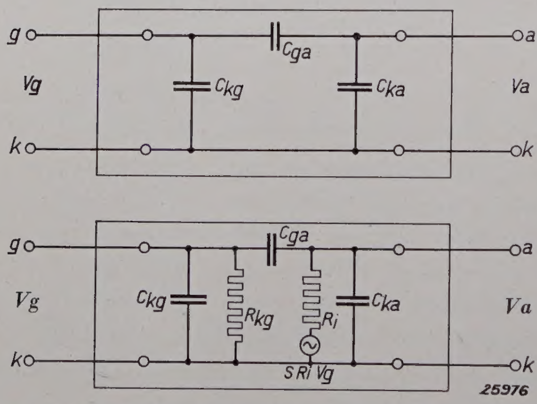

Fig. 1. a) Substitution diagram of a cold amplifier valve. The input admittance is determined by C_{kg}, the output admittance by C_{ka}, slope and reaction are given by the capacity C_{ga}.

b) Substitution diagram of a heated amplifier valve. Currents flow to the grid and to the anode, both are modulated by, and in phase, with the grid voltage. This is expressed by a resistance R_{kg} across the input terminals and a resistance R_i with an E.M.F. of the amount $SR_i v_g$ in series across the output terminals.

When a valve is put into use, grid and anode no longer remain insulated from the cathode, but electrons can pass over from the cathode to the other electrodes. For the input and output admittances, C and B, this means that ohmic conduction is now possible, and thus certain resistances are now in parallel with the condensers. Moreover, the magnitude of the input capacity will change somewhat due to the presence of electrons in the space between cathode and grid, a fact which was noted previously in this periodical [2]). The influence on the slope A and on the reaction admittance D is more difficult to indicate by elements of an electrical circuit. Actually the reaction is scarcely changed, while the slope receives a large ohmic component S_0, due to the fact that the electron current to the anode is greatly influenced by the grid voltage in intensity and phase. The substitution diagram of the valve in the working state is reproduced and explained in fig. 1*b*.

Summarizing, we may now express the four characteristic admittances of an amplifier valve for not too high frequencies as follows:

$$\left.\begin{array}{l}
\text{Slope: } A = j\omega C_{ga} - S_0. \\
\text{Output admittance:} \\
\qquad B = j\omega\,(C_{ka} + C_{ga}) + 1/R_i. \\
\text{Input admittance:} \\
\qquad C = j\omega\,(C_{kg} + C_{ga}) + 1/R_{kg}. \\
\text{Reaction admittance:} \\
\qquad D = j\omega\, C_{ga}.
\end{array}\right\} \quad \ldots \quad (2)$$

In practice the circuit becomes somewhat simpler because of the fact that the control grid is usually made so strongly negative that practically no electrons pass between cathode and control grid. The resistance R_{kg} then becomes infinitely large and $1/R_{kg}$ cancels.

Causes and nature of the dependence of the admittances on the frequency

At wave lengths below the region from about 20 to 40 metres equations (2) for the characteristic admittances are found to be no longer valid. Two reasons for this may be pointed out. One of these has already been noted. We have assumed that the inductive impedances of the leads are small with respect to the capacitive impedances between the electrodes. With increasing frequency, however, the capacitive impedances decrease and the inductive impedances increase, so that the assumption mentioned must be incorrect below a certain wavelength. With the ordinary dimensions of amplifier valves the inductive and capacitive impedances reach the same order of magnitude for wavelengths of a few metres.

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[1]) In this form the substitution diagram holds for a triode. It can, however, also be used for valves with more electrodes, if we assume that all the electrodes except the control grid and the anode are short circuited with respect to the cathode for alternating voltages. Then the symbols C_{kg} and C_{ka} must be taken to represent respectively the capacity between the control grid and all the electrodes connected with the cathode and the capacity between the anode and all the electrodes connected to the cathode.

[2]) C. J. Bakker. Some properties of receiving valves at short waves. Philips techn. Rev. 1, 171, 1936.

The second reason is the finite transition time of the electrons. It has been shown previously [2]) in this periodical that a considerable increase in the size of the real part of the input admittance occurs for frequencies the oscillation time of which is no the transition time of the electrons between cathode and control grid. Further it is clear that in this frequency range the slope will also be affected by the transition time, because the anode current will be later in phase than the grid voltage due to the transition time, so that instead of the real ratio S_0 the ratio between the two must be a complex quantity. For the output admittance and the reaction admittance the influence of the finite transition speed is of less importance, as will be shown later.

In order to be able to deal with all these phenomena in a uniform way, we shall study the general nature of the dependence of an admittance on the frequency.

When the influence of the frequency on the admittances is determined by capacities, inductances and transition times, the frequency can appear only in the formulae in the combination $j\omega$. Except in this combination the imaginary j cannot occur in the formulae for the admittances. We may develop an arbitrary admittance Y in a Taylor series as follows:

$$Y = Y_0 + (j\omega)\, Y_1 + (j\omega)^2\, Y_2 + (j\omega)^3\, Y_3 + \ldots$$

In this formula the quantities Y_0, Y_1, etc. are real. If we now divide Y into a real and an imaginary part:

$$\begin{aligned} Y\mathrm{real} &= Y_0 - \omega^2 Y_2 + \ldots \\ Y\mathrm{imag.} &= j\omega(Y_2 - \omega^2\, Y_3 + \ldots) \end{aligned}$$

We may let the real part of the admittance be equal to the reciprocal of a resistance. The part of this reciprocal resistance which depends on the frequency is then proportional to the square of the frequency in the first approximation. The imaginary part of Y can be considered as being caused by a capacity. The part of this capacity dependent on the frequency is then in the first approximation also proportional to the square of the frequency.

This approximation represents with sufficient accuracy the behaviour of radio valves for frequencies up to 60 megacycles/sec (5 metres). In other words, when the characteristic admittances are given as in equation (2), we find that the capacities and reciprocal resistances are not constant, but each one contains a term, in addition to the constant, which is proportional to the square of the frequency.

The following is a discussion of the four admittances individually.

The slope A

The measurement of the slope is carried out by applying a low alternating voltage to the grid of the amplifier valve to be measured, and by placing between grid and anode an admittance whose real and imaginary parts are variable. By adjusting this admittance so that the alternating current in the anode circuit is zero, which is always possible, the measurement of the slope is reduced to the determination of a complex admittance. It is found that at phase angles between 0° and 90° a detuned circuit can be used as admittance. The measurement of the modulus and the phase angle of this circuit offers no difficulties.

For low frequencies the slope of a valve may be considered real. The imaginary component $j\omega C_{ga}$ (see equation (2)) is very small, especially in screen grid valves, and may be entirely neglected in comparison with the real component of the slope due to the electron current.

With short waves the electron current will need a certain amount of time to pass from the cathode to the anode. A simple estimation of the transition times may be obtained by considering the case of a homogeneous field between two flat plates a and b at a distance d from each other, which have respectively the potentials V_a and V_b with respect to the cathode. The speed v of the electron is determined at every point between the plates by the law of the conservation of energy:

$$\frac{m}{2}\, v^2 = eV.$$

The transition time of the electron between a and b follows from this:

$$t_{ab} = \int_0^d \frac{\mathrm{d}x}{v} = \sqrt{\frac{m}{2e}} \int_0^d \frac{\mathrm{d}x}{\sqrt{V}}\,.$$

Now for a homogeneous field

$$\frac{\mathrm{d}x}{d} = \frac{\mathrm{d}V}{V_b - V_a}$$

and therefore

$$t_{ab} = \sqrt{\frac{m}{2e}}\;\frac{d}{V_b - V_a}\int_{V_a}^{V_b}\frac{\mathrm{d}V}{\sqrt{V}} = \frac{2d\sqrt{\dfrac{m}{2e}}}{\sqrt{V_b}+\sqrt{V_a}}\,.$$

The value of e/m is 17.6×10^{14} cm^2 sec^{-2} volt^{-1}.

Suppose for example that $V_b = 0$, $V_a = 250$ volts, and $d = 0.5$ cm, which would about represent the condition between the suppressor grid and the anode of a pentode; one then finds:

$$t_{ab} = \frac{2 \cdot 0.5\sqrt{1/35.2 \cdot 10^{-7}}}{\sqrt{250}} = 1.07 \cdot 10^{-9}\ \text{sec.}$$

In this way the transition times between all the electrodes can be calculated. The total transition time t_{ka} is obtained by adding. The existence of the transition time, as has already been mentioned, results in the slope taking on a complex value at wave lengths of a few metres, where the frequency is no longer very small with respect to $1/t_{ka}$.

Instead of the real and the imaginary component, we may also consider the modulus and the phase of the slope as functions of the frequency and this description is simpler in this case because it has been found by experiment that the modulus of the slope remains practically unaltered, and the phase angle φ_t due to the transition time changes proportionally with the frequency. It is found that the transition times between different electrodes do not all contribute proportionally to the phase shift.

For a triode it may be calculated:

$$\varphi_t = -\,\omega(0.36\,t_{kg_1} + 2/3\,t_{g_1a}) \quad \cdot \cdot \cdot \cdot \cdot \cdot \quad (3a)$$

and for a pentode:

$$\varphi_t = -\,\omega(0.36\,t_{kg_1} + t_{g_1g_2} + t_{g_2g_3} + 2/3\,t_{g_3a}) \quad (3b)$$

In these formulae t_{kg_1} is the time of transition of the electrons between cathode and control grid, $t_{g_1g_2}$ the time between control grid and screen grid, $t_{g_2g_3}$ the time between screen grid and suppressor grid, t_{g_3a} the time between suppressor grid and anode.

The fact that the coefficient of ωt_{kg_1} is less than one may be explained as follows. If a sinusoidal alternating voltage v_g on the control grid would only determine how much alternating current left the cathode, the current $S_k v_g e^{-j\omega t_{kg_1}}$ would pass through the control grid, and this current is thus behind v_g in phase. But in addition the transition time varies periodically with the time, due to the alternating voltage on the voltage on the control grid, and due to this effect there is also an alternating current through the control grid. This latter current is found to be about 90° in phase ahead of the grid voltage. The result is that the resulting alternating current passing through the grid has a phase angle smaller than ωt_{kg_1}. Calculation gives $\varphi = 0.36\,\omega t_{kg_1}$.

The fact that the coefficient of ωt_{g_3a} is less than one may be explained in the following way. When an electron passes through the third grid, there is an equal positive image charge on the third grid. When the electron moves towards the anode, this image charge flows gradually from the third grid to the anode, so that when the electron arrives on the anode the whole image charge has also just arrived there. In the anode lead, therefore, current is flowing as long as the electron is between the third grid and the anode. If current only flowed in the anode lead when the electron had arrived on the anode, the coefficient of ωt_{g_3a} would have the value one, but since current is flowing as long as the electron is on the way from the third grid to the anode, the coefficient will be smaller than one. An accurate calculation shows that the coefficient must have the value $^2/_3$ when the electron passes the third grid at a low speed. Finally, in order to understand why in the equation for the pentode the terms $\omega t_{g_1g_2}$ and $\omega t_{g_2g_3}$ appear with the coefficient one, we must assume that the electrodes shield each other so well that the current excited by a change in the control grid voltage only begins to flow in the anode lead when the electron is between the suppressor grid and the anode. The transition time between control grid and suppressor grid is then entirely manifested in the phase shift.

The transition time, however, is not the only cause of a phase shift between the anode current and the grid voltage. It is obvious that the self-inductance of the cathode lead also has a similar effect. When there is an alternating voltage v_g between the grid and the external terminal of the cathode, the following current flows through the cathode lead:

$$i_k = S_k\,v_g.$$

This current experiences an inductive reactance in the lead which corresponds to a inverse e.m.f.

$$i_k\,j\omega\,L_k = j\omega\,S_k L_k v_g.$$

The voltage between the grid and the cathode itself thus becomes

$$v_g' = v_g\,(1 - j\omega\,S_k L_k) \quad \cdot \cdot \cdot \cdot \quad (4)$$

and when $\omega S_k L_k \ll 1$, this means that the anode current lags a phase angle $\omega S_k L_k$ behind the external applied modulation voltage.

The total phase shift is obtained in fair approximation by adding the two results. Thus for a triode:

$$\varphi = -\,\omega\,S_k L_k - \omega\,(0.36\,t_{kg_1} + 2/3\,t_{g_1a}) \quad \cdot \quad (5a)$$

and for a pentode:

$$\varphi = -\,\omega\,S_k L_k - \\ -\,\omega\,(0.36\,t_{kg_1} + t_{g_1g_2} + t_{g_2g_3} + 2/3\,t_{g_3a}) \quad \cdot \cdot \quad (5b)$$

Equations (5a) and (5b) were tested by measurements on a pentode of the type AF7. The values measured at a wave length of 9.1 m are:

as pentode with anode current of 3 mA:

$$|S| = 2.69 \text{ mA/V}; \; \varphi = -22^\circ;$$

as triode with 4 mA anode current:

$$|S| = 3.82 \text{ mA/V}; \; \varphi = -\ 6^\circ.$$

In the measurement as a triode, anode, suppressor grid and screen grid were interconnected; in the measurement as a pentode, screen grid and suppressor grid were connected to the cathode through large condensers. For a very low frequency we found $S_0 = 2.7$ mA/V for the pentode connection, and $S_0 = 3.8$ mA/V for the triode connection.

It may be noted in the first place that the modulus of the slope at 9.1 m is equal to the static value within the limits of experimental error. The phase angles measured must now be compared with the values which can be calculated from equations (5a) and (5b). We found:
$S_k = 3.8$ mA/V and L_k about 5×10^{-8} H, and the following values:

$$\begin{array}{lcll}
\text{Triode} & & \text{Pentode} & \\
t_{kg_1} & = & t_{kg_1} & = 0.77 \cdot 10^{-9} \text{ sec,} \\
t_{ga} & = & t_{g_1 g_2} & = 0.26 \cdot 10^{-9} \text{ sec,} \\
 & & t_{g_2 g_3} & = 0.49 \cdot 10^{-9} \text{ sec,} \\
 & & t_{g_3 a} & = 0.75 \cdot 10^{-9} \text{ sec,} \\
S_k L_k & = & S_k L_k & = 0.19 \cdot 10^{-9} \text{ sec.}
\end{array}$$

From this it follows at 9.1 m wave length:

	measured	calculated
as triode	— 6°	— 7.5°
as pentode	— 22°	— 20.5°

This good agreement was also found with measurements on other valves, so that the explanation given for the phase angle of the slope may be considered complete. At the same time this supplies proof that the dependence on frequency of the phase angle of the slope is well represented by formulae (5a) and (5b).

The significance of the phase angle of the slope

The phase angle of the slope is important in all those cases in which a portion of the amplified voltage is fed back to the input circuit in order to diminish its damping or to excite oscillations. If the feed back voltage is not in phase with the input voltage, not only is the damping of the input circuit affected by the feed back, but also its resonance frequency. A large phase angle of the slope in the case of an oscillator may therefore unfavourably affect the frequency stability with variations of the supply voltage.

The phase angle of the slope may also be important in cases where feed back is not deliberately employed, for example, in high frequency amplifiers. Due to the capacity between control grid and anode an (undesired) back coupling may also occur in this case. The phase angle of the slope can then increase the tendency to oscillation of a high frequency stage.

The output admittance B

The measurement of the output admittance is in principle as follows. The output side of a valve (cathode, anode) is connected in parallel to a circuit whose resonant impedance is known. The circuit is then detuned by the capacitive part, and damped by the ohmic part of the output admittance. From the detuning the output capacity can be determined, and by tuning the circuit anew, and determining the new damping, the output parallel resistance is obtained.

As a result of such determinations it is found that the output capacity hardly changes with the frequency, while the real part of the output ad-

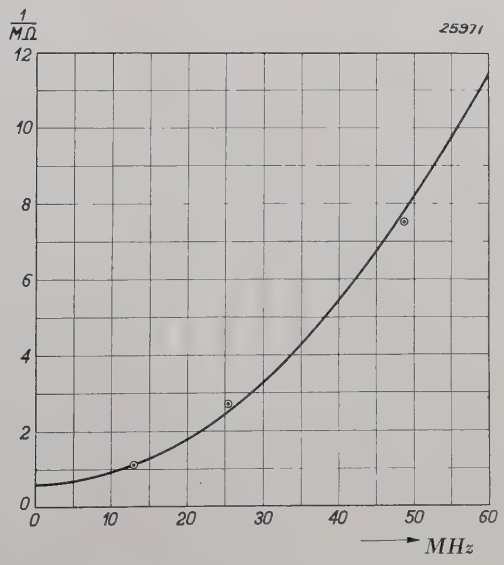

Fig. 2. The real part of the output admittance of a valve EF6 plotted against the frequency. Points measured. The curve drawn is given by the formula:
$\frac{1}{R_i} = (0.60 + 0.30 \cdot 10^{-2} \nu^2)$ micromhos ,where ν is in megacycles per sec.
It may be seen that the points fall very well on the second order curve. For low frequencies E is equal to the internal resistance of the valve which is of the order of magnitude of 2 MΩ.

mittance as a function of the frequency above 10 megacycles/sec (30 m) increases considerably, namely proportionally to ω^2, as may be seen from the results of measurement on a pentode EF6 given in fig. 2. The output parallel resistance of the valve EF6 is 2 MΩ at low frequency, and at 6 megacycles/sec (5 m) it is only 5000 Ω.

In order to find out whether the transition times of the electrons in valves of a type like EF6 are important for the output admittance on short waves, we shall consider what happens in the space between suppressor grid and anode. We apply a low alternating voltage v_a to the anode, and wish to determine the alternating current i_a to the anode caused hereby. When we consider the internal resistance as infinitely great, only direct current passes through the suppressor grid. Between the suppressor grid and the anode, however, an alternating current is found to act because of the fact that the transition time varies with time. It is clear that the admittance i_a/v_a corresponding to this is proportional to the direct current passing through the suppressor grid. If thus the admittance contains a real part, and therefore represents a damping, this must also be proportional to the current.

This has been tested by experiment. The dimensions of the third grid and of the anode are exactly the same in the pentodes AF3 and AF7. The output damping of the two valves also varies only slightly. At 8.0 m the output damping of AF3 is 7.7 micromhos, and of AF7 is 8.7 micromhos. The difference between the anode currents, however, is much greater. With AF3 the anode current at the working point is 8 mA, and with AF7 only 3 mA. From this it may be seen that the transition time of the electrons between the third grid and the anode cannot play an important part in the anode parallel resistance.

There must, therefore, be another cause. The conductance which corresponds to the statically measured internal resistance is only about 0.5 micromhos for both valves, and thus need not be considered. The greater part of the output damping comes from the self-inductances and mutual inductances of the leads and the capacities between the electrodes of the valve. If an alternating voltage is applied to the anode, alternating currents flow through the capacities between anode and suppressor grid, anode and screen grid, etc. to screen grid and suppressor grid, and then through the leads to earth. Due to the mutual induction of these leads with the cathode connection, a slight alternating voltage is caused between the cathode lead in the valve and the cathode terminal, and this alternating voltage also acts between control grid and cathode. In an amplified form it is returned to the anode in the valve, and this may cause damping. Measurements of this effect have been carried out with an AF3 valve in which the capacity C_{g_3a} between anode and third grid, and the mutual inductance M_{kg_3} between the connections to third grid and to cathode are principally concerned. From the above reasoning the following formula may be set up:

$$\frac{1}{R} = \omega^2 \, S \, C_{g_3a} \, M_{kg_3} \, ,$$

where S is the slope of the tube.

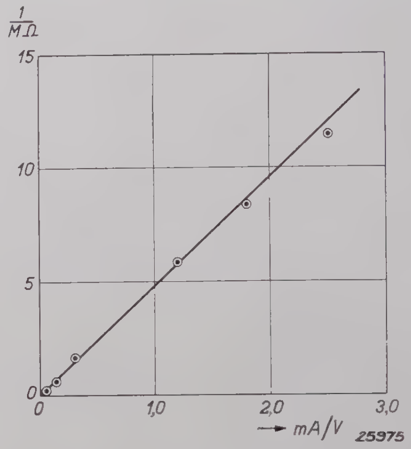

Fig. 3. Values of $1/R$ as a function of the slope S in mA/V for the valve AF3. As required by the theory the measured points (circles) lie on a straight line. Wave length 8.0 m.

In fig. 3 the values of $1/R$ for the valve AF3, measured at a wavelength of 8.0 m, are plotted against the slope S of the valve. From the linear relation between $1/R$ and S, and under the reasonable assumption that $C_{g_3a} = 3.5$ μμF, the value 24×10^{-9} H follows for M_{kg_3}. This is a probable order of magnitude. This also explains why valves AF3 and AF7 have approximately equal anode parallel resistances, because they have the same values of C_{g_3a} and M_{kg_3}, while the slopes also differ very little at the working points.

The input admittance C

The measurement of the input admittance is carried out on the same principle as that used for measuring the output admittance.

Fig. 4 gives the result of such a determination. The real part of the input admittance is plotted in double logarithmic coordinates against the frequency. This curve shows that the input parallel resistance so defined varies proportionally with the square of the wave length up to very short waves, which proves that higher powers of ω play practically no part in the development of the series.

The real part of the input admittance may here

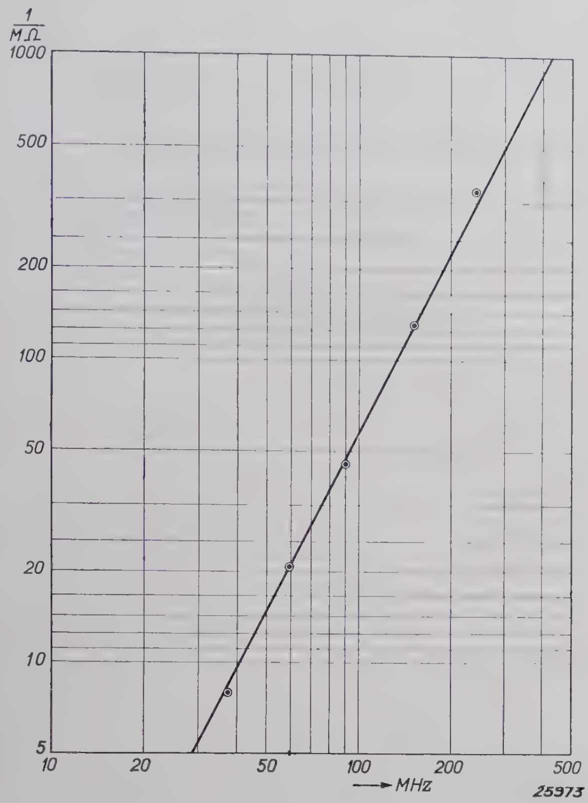

Fig. 4. Input parallel resistance of a pentode as a function of the frequency. The frequency in megacycles/sec is plotted horizontally, and the input damping in micromhos vertically. It may be seen that the input parallel resistance varies as the square of the frequency.

also be ascribed to two essentially different causes, and consists of two terms $1/R_t$ and $1/R_{LC}$, which come respectively from the transition time of the electrons, and from the inductances in the valve together with the capacity between grid and cathode:

$$\frac{1}{R_{kg}} = \frac{1}{R_t} + \frac{1}{R_{LC}}\,.$$

For the transition time part, the following formula has already been derived in this periodical [2]):

$$\frac{1}{R_t} = \frac{1}{20} f \cdot S_k \cdot (\omega t_{kg})^2 \quad . \; . \; . \; . \quad (6)$$

In this formula t_{kg} is the transition time of the electrons between cathode and control grid, and f is a factor, usually of the order of 2, which depends upon the transition time of the electrons between control grid and anode (or screen grid, when we are concerned with tetrodes or pentodes).

Upon consideration of the influence of the different inductances on the real input admittances it is found that the self inductance L_k of the cathode lead plays the most important part. For the contribution to $1/R_{kg}$ of this self inductance, the following formula is found:

$$\frac{1}{R_{LC}} = \omega^2 S_k L_k C_{kg_1}, \quad . \; . \; . \; . \; . \quad (7)$$

where C_{kg_1} is the capacity between control grid and cathode.

Equation (7) may be derived as follows. As showns above (equation (4)), the voltage between cathode and grid differs in phase from the external applied modulation voltage and is:

$$v_g' = v_g\,(1 - j\omega S_k L_k). \quad . \; . \; . \; . \quad (4)$$

This voltage is connected with the capacity C_{kg_1} between grid and cathode. The current to the grid is therefore

$$i_g = v_g\,(1 - j\omega S_k L_k)\, j\omega C_{kg_1},$$

from which it follows that an input admittance is given of

$$\frac{i_g}{v_g} = j\omega C_{kg_1} + \omega^2 S_k L_k\, C_{kg_1}\,,$$

in agreement with equation (7).

Separation of the influences of transition times and inductances on the input admittance

Formulae (6) and (7) for the input parallel resistance caused by the transition times and inductances correspond closely. In order to demonstrate this agreement even more clearly let us consider the oscillation time T_{kg_1} of the alternating current for which the self-induction of the cathode lead and the capacity between cathode and grid are in resonance. This time of oscillation is given by:

$$T_{kg_1} = 2\,\pi \sqrt{L_k\, C_{kg_1}}$$

and by inserting this value into equation (7) it follows that

$$\frac{1}{R_{LC}} = \frac{1}{4\,\pi^2}\, S_k\, (\omega T_{kg_1})^2,$$

which formally resembles very much the transition time effect:

$$\frac{1}{R_t} = \frac{f}{20}\, S_k\, (\omega t_{kg_1})^2\,.$$

As already mentioned the resonances in the valve usually appear at wave lengths between 1 and 2 m, therefore with oscillation times of 3 to 6×10^{-9} sec. Now the transition times of the electrons between cathode and grid are just of the same order of magnitude so that the two causes of damping will be practically equally important.

An experimental separation of the two influences on the input damping is possible by comparing the damping of a single system with that of two

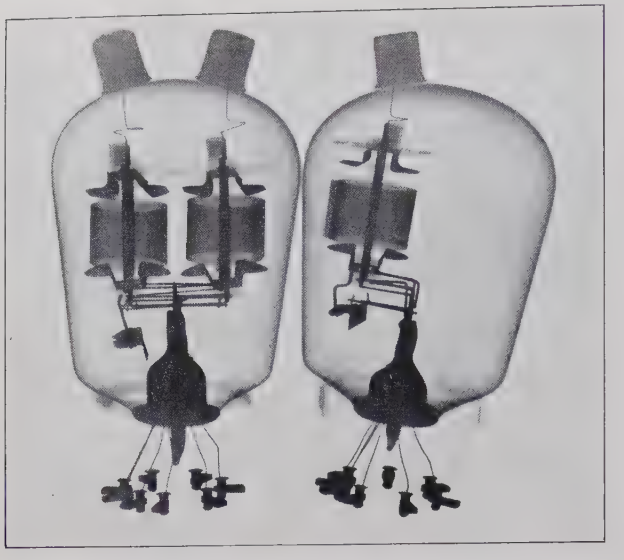

Fig. 5. X-ray photograph of a single and a double pentode (type EF5) for separating the two causes of the input damping on short waves.

systems connected in parallel with common leads (see X-ray photographs *fig. 5*).

Upon doubling the valve, the slope is doubled and so is the $1/R_t$ value, since the other quantities in equation (6) remain unaltered. The induction part (equation (7)) is, however, quadrupled, because it is not only proportional to the slope, but also to the capacity C_{kg_1} which is also doubled. If we call the input parallel resistance of the single and of the double valve R_e and R_d respectively, then:

$$\frac{1}{R_e} = \frac{1}{R_{LC}} + \frac{1}{R_t},$$

$$\frac{1}{R_d} = \frac{4}{R_{LC}} + \frac{2}{R_t},$$

and from this follow the two input parallel resistances R_{LC} and R_t:

$$\frac{1}{R_{LC}} = \frac{1}{2R_d} - \frac{1}{R_e},$$

$$\frac{1}{R_t} = \frac{2}{R_e} - \frac{1}{2R_d}.$$

Table I gives some results, obtained in this way.

Table I. Measurements of the input damping of three test valves at a wave length of 7 m.

Valve No.	distances between electrodes		S_k	R_t	R_{LC}	$\frac{1}{R_{LC}} : \frac{1}{R_t}$
	k-g_1	g_1-g_2				
	mm	mm	mA/V	Ω	Ω	
1	0.10	0.30	18	60000	600	100
2	0.20	0.76	2.3	27000	46000	0.60
3	0.35	2.10	1.9	11000	39000	0.28

With increasing distance between the electrodes the share of the transition times in the total damping becomes steadily greater because the transition time increases and the capacity decreases.

Valve No. 1 has in connection with its very steep slope an unusually long cathode which has a relatively great capacity with respect to the control grid; for this reason, and due to the steepness of the slope, the damping resistance R_{LC} is much smaller here than in ordinary amplifier valves.

Significance of the input impedance

In the case of an amplifier stage, in which a tuned circuit is included in the grid connection, the input parallel resistance acts across the circuit, which is thereby damped and the amplification may be considerably decreased. When several similar amplifier stages are connected in series, the input resistance of the following valve always acts across the circuit connected to the anode. The output resistance of the valve is connected in parallel with this. This latter resistance, however, is very much greater than the input resistance at all frequencies, so that it has practically no influence on the damping. The greatest possible resistance in the anode connection is thus the input parallel resistance R_{kg}, and the greatest possible anode alternating voltage thus becomes:

$$v_a = i_a R_{kg} = v_g S R_{kg}.$$

At very high frequencies the product SR_{kg} may become less than one; no amplification is then possible. This shows how important it is to have the input resistance as high as possible. Because of this for very short waves valves have been designed with very small dimensions and very short connections to the different electrodes. In this way the influence not only of transition times but also of induction is avoided as much as possible. The measurements shown in fig. 4 refer to such a valve. Considering the fact that this valve has a slope of 1.5 mA/V, it follows from fig. 4 that at a wave length of 1 m an amplification by a factor 2 can still be attained.

The reaction admittance D

The reaction admittance can be measured by applying a known alternating voltage to the anode, and connecting a tuned circuit of known impedance between control grid and cathode. From the measurement of the alternating voltage across this circuit the reaction admittance can be derived.

Measurements of the reaction admittance show no important difference between a cold and a warm tube. This shows that the electrons are scarcely concerned with this admittance. It is found moreover that this admittance remains purely imaginary even at very high frequencies, and in agreement with our general considerations it can be represented by

$$D = j\omega C_{ga}' = j\omega(C_{ga} - K\,\omega^2), \quad \cdot \;\cdot \;\cdot \quad (8)$$

where K is usually positive. K depends upon the self- and mutual inductions of the leads, and upon the valve capacities.

In *fig. 6* the value of C_{ga}' measured for a pentode AF3 is plotted against the frequency. A curve of the second order has been drawn as well as possible through the points measured; it may be seen that the agreement is good.

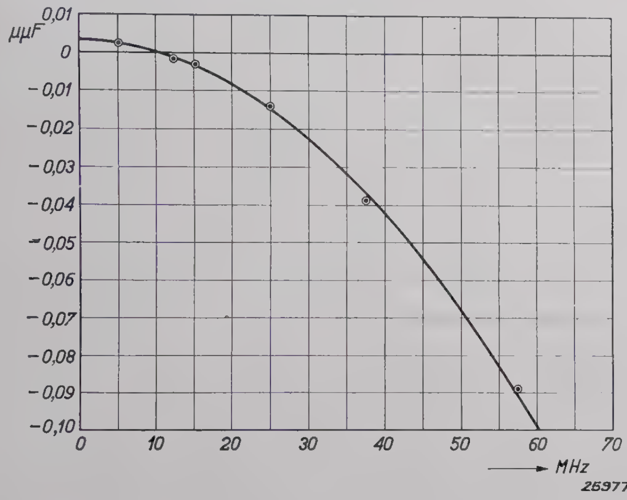

Fig. 6. The capacity C_{ga} of an AF3 valve plotted as a function of the frequency. Points measured. Curve drawn is given by the formula: $C_{ga}' = (0.0030 - 0.285 \times 10^{-4}\nu^2)$ μμF, where ν is in megacycles per sec. It may be seen that the measured points fall satisfactorily on the second order curve in agreement with formula (8)).

The variable part of the reaction admittance may be explained as follows. When a high frequency alternating voltage v_a acts on the anode, currents flow to the different electrodes through the capacities C_a of the valve. Due to the self inductances and mutual inductances of the leads, alternating voltages are found to act on these electrodes, and due to the capacity C_g between these electrodes and the grid, currents flow to the grid. If we call the average inductance (self-induction or mutual induction) of the leads concerned in this process M, the current may to the first approximation be written as follows:

$$i_g = v_a \cdot j\omega C_a \cdot j\omega M \cdot j\omega C_g = v_a \cdot j\omega\,(-C_a M\,C_g\omega^2),$$

which corresponds to equation (8).

That this representation is correct was proved by the fact that it was possible, by introducing a suitably chosen mutual inductance between the connections to the screen grid and to the anode, to compensate the term $K\omega^2$ completely. The arrangement of the circuit for this purpose is drawn in *fig. 7*.

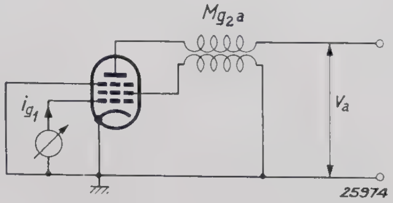

Fig. 7. Compensation of the reaction on ultrashort waves by means of a mutual induction M_{g_2a} between anode lead and screen grid lead. M_{g_2a} is chosen so that the ω^2 term in formula (8) is entirely compensated.

Equation (8) is an approximation which is only valid outside the region where resonances in the valve become pronounced. In this region the reaction admittance may become very large. In order to make this plain a substitution diagram is given in *fig. 8* for the case in which the self-inductance L_{g_2} of the connection to the screen grid is in resonance with the capacity C_{g_2} between the screen grid and the other electrodes. In this case a very high alternating voltage is found to act on the screen grid, so that, due to the capacity between screen grid and control grid, a very high current will flow to the control grid.

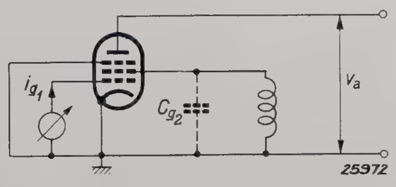

Fig. 8. The appearance of resonances in the valve at high frequencies. The self inductance L_{g2} of the screen grid lead is tuned with the capacity C_{g2} of the screen grid to the frequency of v_a. This is expressed in a strong current i_{g1}, i.e., in a high conductivity from anode to the first grid.

The reaction admittance is undesirable in a high frequency amplifier valve and must be kept as small as possible. It is by the reaction admittance that a part of the amplified voltage is conducted back to the grid. If this back coupling becomes too great, oscillation can occur in certain circumstances.

A CAR RADIO

by J. W. ALEXANDER.

621.396.62 : 629.113

The most important demands made on a motor car radio set are: small dimensions, high sensitivity, shielding against external interferences, ease of installation, very great ability to withstand shocks, low current consumption and large output of sound. In this article it is shown how these partially conflicting requirements can be satisfied by constructing a set in which use is made of parts especially designed for this purpose, including such as radio valves, loudspeaker and arrangement for operation.

Introduction

In recent years there has been a steadily increasing interest in radio sets for the motor car. These receiving sets are built specially for use in a car and are capable of reproducing the programmes of the ordinary broadcasting stations. A receiving set such as that ordinarily used at home cannot be used in a car for various reasons. The circumstances under which the sets work, the consequent difficulties and their solutions will be discussed in the following, from which it will also appear why a car radio set differs so much from a radio set for use at home.

In general there may be said to be five points which exert an influence on the construction of car radio sets. These points are connected with:

1) Aerial,
2) Loudspeaker,
3) Supply,
4) Interferences,
5) Mechanical construction.

The aerial

An aerial for a car radio set for obvious reasons can only have a limited height in comparison with an ordinary house aerial; because of this the voltage excited in the aerial by the transmitter field will be relatively small. The effective height (cf. Philips techn. Rev. **2**, 216, 1937), which in the case of a house aerial is about 1 m on the average, is only a few centimetres in the case of a car radio. The result is that the sensitivity of the motorcar set must be several times as great as that of a house set, in order to be able to receive ordinary broadcasting stations satisfactorily.

In addition, the signal entering the set will be further considerably weakened due to the cable connection between aerial and set. This connection must be well shielded in order to prevent the electrical equipment of the car from interfering with reception. If the aerial has a capacity C_a to earth, and the connection cable a capacity C_k, a voltage occurs between the input terminals of the set which is proportional to $\frac{C_a}{C_a + C_k}$. From this it may be seen that the capacity of the cable must be as small as possible and the capacity of the aerial as large as possible.

However, the development of the motorcar aerial has proceeded in a direction such that the capacity of the aerial has become steadily smaller. Formerly the aerial usually took the form of a strip of wire gauze introduced under the roof covering. These aerials had a capacity of about 150 cm and higher. The introduction of the steel top has made this form of aerial impossible, and various other forms have appeared, such as the aerial under the car in the form of a rod or wire, the aerial on the top, and at present also the aerials of constant or variable length in the form of rods which are fastened to the side of the car or on the bumper. The capacity of these aerials is only of the order of 30 cm, while the capacity of a house aerial is 200 cm on the average.

Due to the low intensity of the signal received it is necessary as was stated above, to increase the sensitivity of the receiving set many times. The sensitivity of a receiving set is defined in the following way. A high frequency voltage with a low frequency modulation of 30 per cent and 400 cycles acts in the aerial circuit. This voltage is transformed by the apparatus into low frequency energy, which, for measurement of sensitivity, is not conducted to the loudspeaker, but to a suitably chosen ohmic resistance. 1 W is usually chosen as the normal value of this low frequency power for automobile radios. The sensitivity of the set is then the number of microvolts necessary in the aerial to obtain this low frequency energy of 1 W in the above-mentioned output resistance. For automobile radio sets the sensitivity is of the order of some microvolts; for a house radio for the same low frequency energy a signal of 50 to 100 microvolts in the aerial is necessary, so that it may be said that an automobile radio set is about 20 times more sensitive than an average house set.

The increase of the sensitivity is limited by the noise occurring in the receiver itself (see Philips techn. Rev. **2**, 136, 1937). As is shown in the article quoted, because of the thermal fluctuation of the electricity in a conductor with resistance R, a voltage appears such as that given by the following formula:

$$\overline{V^2} = 4\ kTR\ \Delta\nu,$$

where V^2 represents the contribution to the mean square of the voltage for the frequency interval between ν and $\nu + \Delta\nu$, k Boltzmann's constant and T the absolute temperature. A tuned LCR circuit may in general, in the frequency region in the neighbourhood of its tuning point, be considered as a resistance of the value L/CR. For the width of the frequency region around the tuning frequency, which is amplified by the apparatus, we may take as an approximation 6000 c/s. The value R/L, which determines the shape of the resonance curve of the input circuit, is, under normal circumstances, 120 000 c/s. At a value of C of 50 $\mu\mu$F of the tuning condenser, and for a temperature of 300° K we therefore find:

$$\overline{V^2} = 4 \cdot 1.37 \cdot 10^{-23} \cdot 300 \cdot \frac{1}{50 \cdot 10^{-12} \cdot 12 \cdot 10^4} \cdot 6 \cdot 10^3$$

$$\overline{V^2} = 16 \cdot 10^{-12},$$

so that the square root of the mean square of voltage is:

$$\sqrt{\overline{V^2}} = 4\ \mu\text{V}.$$

The noise thus gives as much low frequency energy as a signal voltage on the first circuit of 12 μV with an average modulation of 30 per cent.

From this it follows that there will be a signal strength below which the noise becomes disturbing; at higher signal strengths the ratio of signal to noise is more favourable. This noise effect can appear sooner in a car radio set than in a house set, because the car set is extremely sensitive and the circumstances can change so completely while the car is moving. When the car is in the open, the transmitter field will be much stronger than when the car is passing through a narrow street between high buildings, or, even worse, when it is within the metal structure of a bridge where it is practically surrounded by a Faraday cage. This difference in field strength is practically removed by the automatic volume regulation which provides that the amplification of the apparatus shall depend upon the strength of the signal received. The amplification is greatest with the weakest signal, and decreased with increasing strength of the signal. In this way the acoustic energy delivered remains almost constant. When the circumstances are very unfavourable (Faraday cage) the amplification becomes higher, so that the noise effect begins to be clearly heard.

The loudspeaker

In a moving motorcar the loud speaker must give more output than in a room in a house. While in the room the music is sufficiently loud when the level of sound is 50 phons (see Philips techn. Rev. **2**, 54, 1937), it can scarcely be appreciated in an automobile whose own level of sound varies between 45 and 65 phons. Therefore a much higher level is necessary depending on the construction of the car, the running speed, the wind, the nature of the road surface, etc.

In addition to the necessity of establishing a high level of sound because of the presence of other sounds, the absorption of sound inside the car also plays a part. In this factor, as is known from acoustics, the upholstery of the car is very important, especially with respect to the absorption of high tones. Further the degree of absorption depends upon the number of persons in the car, while the great influence of open windows, especially during running, will immediately be clear.

Thus for a car radio a loudspeaker is necessary which, in spite of its small dimensions, has a high acoustic yield. This is achieved by making a small magnet of a special kind of steel ("Ticonal"), which gives a very high field strength.

Power Supply

Like all other radio sets the car radio needs various different voltages, namely cathode, anode and grid voltage. As a source of energy in the automobile there is the accumulator for the ignition and the other electrical equipment. The cathodes can be connected directly to the accumulator, a much higher voltage than 12 volts (6 volts in some cars) is necessary for the anode voltage, so that an arrangement is necessary for transforming the direct voltage of 12 volts into a direct voltage of about 250 volts. In the beginning of car radio development use was made of separate batteries for the anode voltage, later rotating converters were also used. These installations, which occupied much space and were very expensive, have been practically superseded by the vibrator converter, the principle of which has already been described in this periodical (see Philips techn. Rev. **2**, 346, 1937). In the case there discussed, however, a direct voltage of

100 - 250 volts had to be converted into 100 - 250 volts alternating voltage. In the converter for the car radio the construction is some what different, since one begins here with a direct voltage of 6 volts, while the energy which must be handled by the vibrator is much smaller than in the case considered previously. The principle is given in *fig. 1*. By means of an electromagnet (M) a spring is kept vibrating, and alternating closing contacts (K_1 and K_2). A direct current flows for a time (as long as the contact is closed) alternately through the two halves of the primary winding of the transformer. These currents cause alternately in the two halves of the secondary winding of the transformer, transformed currents which are rectified and smoothed by means of the rectifier valve and the condenser, until there is a direct current of about 250 volts between the terminals C and D.

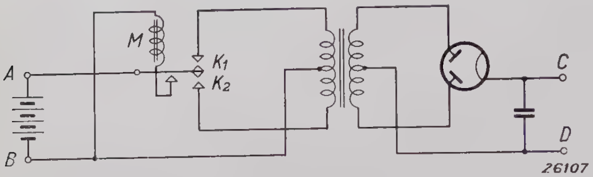

Fig. 1. Diagram of the supply of a motorcar radio set by means of a vibrator converter. The 6 or 12 volts accumulator is connected between the terminals A and B, while the anode voltage of about 250 volts is taken off between C and D. M is the electromagnet which keeps the vibrator moving; the contacts K_1 and K_2 are alternately made and broken.

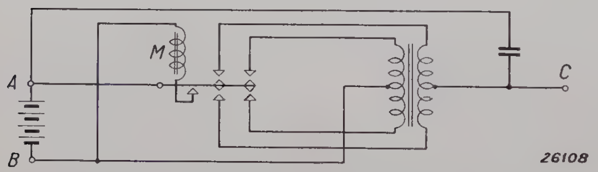

Fig. 2. Construction of the vibrator converter with a second set of contacts, whereby the current direction is reversed simultaneously in the primary and in the secondary winding of the transformer. This makes the use of a rectifier unnecessary. The anode voltage is taken off between the terminals A and C.

In the newest type (*fig. 2*) the rectifier valve is replaced by an extra set of contacts on the vibrator, which are in the secondary circuit and can be moved synchronously with the contacts in the primary circuit. The anode voltage is thereby obtained between the points A and C. This construction has the advantage of greater efficiency and of needing less space because of the omission of the rectifier valve. The whole apparatus thus becomes smaller.

Accumulator supply means in the end that the dynamo which changes the accumulator is more heavily loaded by the car radio. The current consumption of the set must therefore be kept as low as possible. One way of doing this is by keeping the consumption of heating current low.

In addition to this requirement, the valves for the car radio set must also satisfy several other requirements, which together have laid the foundations for a whole new series of valves, the so-called E-series.

These valves are intended for a supply voltage of 6 volts. This is, however, not a constant voltage, but in unfavourable cases may run up to 8 volts, depending on the state of the accumulator, on the dynamo and on the running speed. Because of this, in the supply systems described, not only the cathode voltage but also the anode and grid voltages increase so that the valves are extra heavily loaded. This has been taken into account in the construction of the valves.

Care must not only be taken to provide for a low heating current consumption (with these valves 200 mA), but also for small dimensions. This requirement is satisfied by a very compact construction; the total height of these valves is 90 mm and the greatest diameter 32 mm. In addition by means of careful construction, such as the introduction of extra supports, care has been taken that the valves are very resistant to the shocks and vibrations which they experience in the set while the automobile is running.

Interferences

A very important difference between a radio set for use at home and a motorcar is that the latter is situated in the immediate neighbourhood of a very strong source of interference. These interferences are caused by the ignition system, the ignition works on the following principle (*fig. 3*).

The accumulator is connected via the primary winding of a transformer T (the ignition coil) in series with an intermittent contact K. Due to

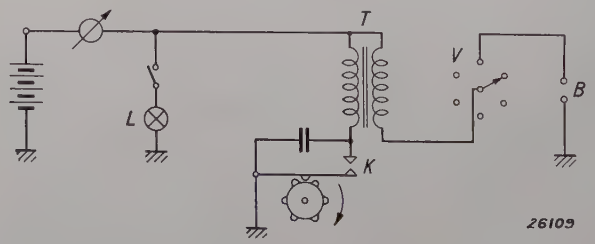

Fig. 3. Diagram of the ignition system. K is the "make and break" contact. The transformer T is the ignition coil, and V is the distributor through which the current is led to one of the sparking plugs B. L is the lighting circuit of the car which also is connected to the accumulator.

this, on the secondary high voltage side of the transformer, surges of the order of 10 000 volts occur, which are conducted over a distributor V successively to the different sparking plugs B. There are thus sudden variations in voltage in the primary as well as in the secondary circuit. If the variation in voltage is divided up into a Fourier frequency spectrum all frequencies are found to be present. With increasing frequency the amplitude increases up to about 30×10^6 cycles per sec and decreases again for higher frequencies. These voltages are not only present on the high and low voltage connections of the ignition system, but since the latter are connected with the accumulator, they also act on all cables connected with it, such as those for the lights (L). Not only do these voltages act on the connections, but also on all other metal parts such for example as the chassis, since this is used as return connection. To reduce these interferences it was customary to connect resistances of the order of 10 000 ohms in the sparking plug line. This decreases the interference very much. However, in the design of the ignition system no account had been taken of a possible later introduction of these resistances, and difficulties arose. The method of complete shielding has also been applied, in which the whole ignition system was placed in an electrically closed space. This is very expensive and complicated, since practically every kind of automobile is constructed differently.

A closer study of the interference has indicated a very much simpler solution: When interferences occur in the case of an ordinary radio, they usually appear in the following manner. The set when tuned to a definite transmitter, is sensitive practically only to voltages of the same frequency as the transmitter. In general an interference will include a whole frequency range. The apparatus will pick out only that frequency to which it is tuned, and this will cause a voltage on the first circuit which is amplified further by the apparatus just like a signal. If we now connect a tuned circuit in series between aerial and earth, it will only be able to diminish the interference at the signal frequency; thus the signal disappears together with the interference.

In a car radio set the situation is more complicated. If here we connect a tuned circuit in parallel with the set the signal disappears but the interference continues. The interference thus occurs because of frequencies other than that of the signal. This is possible here because:

1) the distance between the source of interference and the receiver is so small,
2) the source of interference is so unusually strong.

In a car radio voltages with frequencies greater than those of the broadcasting band enter the receiver with high intensity and then cause interferences due to non-linear phenomena.

For removing such interferences, therefore, the set must be so constructed that the frequencies outside the broadcasting range cannot penetrate into it, while an extra provision is necessary for the frequencies within the broadcasting range. The first is attained by:

1) Making the apparatus itself electrically completely closed (apart from holes for ventilation). This is done by giving the set a metal jacket and closing the necessarily removable parts (for instance for the changing of valves) with springs,
2) Providing the connections entering the set, such as aerial and supply connections, with filters, so that the aerial connection only passes frequencies of the broadcasting range.

In order to keep out the remaining interferences with frequencies in the broadcasting range, it is usually sufficient to introduce a condenser between the chassis and the accumulator side of the ignition coil. Further, in installing the set, care must be taken that the aerial is as far as possible away from interference-bearing conductors, such as for example lamp wiring and battery. In this respect an aerial on the top is to be preferred to one under the car.

Although the ignition system causes the most serious interferences, it is not the only cause of interference. The dynamo for charging the battery also gives appreciable interferences. The introduction of a condenser between the dynamo and the chassis gives satisfactory improvement.

There is an additional group of interferences which do not always appear but which may nevertheless sometimes be very disturbing. The so-called "axle interferences" belong to this group of specific motorcar radio interferences. In a car with rear drive the front wheels turn about an axle which is directly connected with the chassis. Between the axle and the bearing of the wheel there is a thin layer of grease which insulates the wheel from the axle. Especially with older cars there is a possibility that, depending on the shocks experienced by the wheel, more or less good electrical contact will be made between wheel and axle at this point. Crackling interference noises result from this varying contact. The remedy is either a permanent insulation, or a permanent contact which can be obtained by means of a pressure spring between the revolving hub of the wheel and the stationary extremity of the axle. In automobiles with front-wheel drive it is obvious that these springs must be introduced on the rear wheels.

Another incidental interference, the "brake interference" may occur due to the generation of static electricity by the friction of a brake shoe on a brake drum. The remedy for this is the correct adjustment of the brake shoes.

In some cases a "tyre interference" may be experienced which appears to be due to frictional phenomena between the tyre and the road. This interference depends upon the nature of the road surface and appears chiefly with particular kinds of tyres. On a macadam road it is much less disturbing than on concrete roads. Better earthing of the wheel by means of the above-described spring sometimes brings improvement. For this group of interferences also an aerial is to be preferred which is as far as possible away from the source of interference, i. e., on top of the car or on the side.

Mechanical construction

In its mechanical construction a radio set also differs to an important degree from a house set. The requirements made of an auto- mobile radio in this respect are of course quite different. We shall go into several of these:

1) Since the apparatus is installed in the motorcar at a point where the space is very limited as a rule, it must be made as small as possible (*fig. 4*). This is done by using the smallest possible number of parts, each of which is as small as possible, and by placing them as close together as possible, as may be seen from the photograph in which one of the side walls has been cut away (*fig. 5*). In the middle of the foreground two valves have been removed in order to show the part of the apparatus lying behind them. As may be seen from the photograph, the loudspeaker is built into the apparatus. The least amount of space is occupied in this way, and in most cases this apparatus can easily be installed under the dashboard. If still better quality is desired, or when it is necessary to set up the loudspeaker in some other place, as may be the case in a motorbus, a model with a separate loud speaker (*fig. 6*) may be chosen. The loud speaker of this type is larger and more sensitive than is possible with a loud speaker built into the set. For a motorbusses there is also the possibility of connecting a microphone to the set. By means of a reversing switch the conductor may then interrupt the radio broadcast and use the set and the loudspeaker hanging above the driver in order to address the occupants of the bus.

2) The apparatus must be able to be operated from a particular seat, for example from the driver's seat. This is only possible when the parts necessary for operation are fastened on or under the dashboard or on the column of the steering wheel. Since, however, there is usually no space available at these points for installing the set itself, it is necessary to introduce remote control. The parts necessary for operation are then assembled in an operating box (*fig. 7*) which can be introduced under (*fig. 8*) or, by means of another housing, in the dashboard (*fig. 9*). The right-hand knob of the

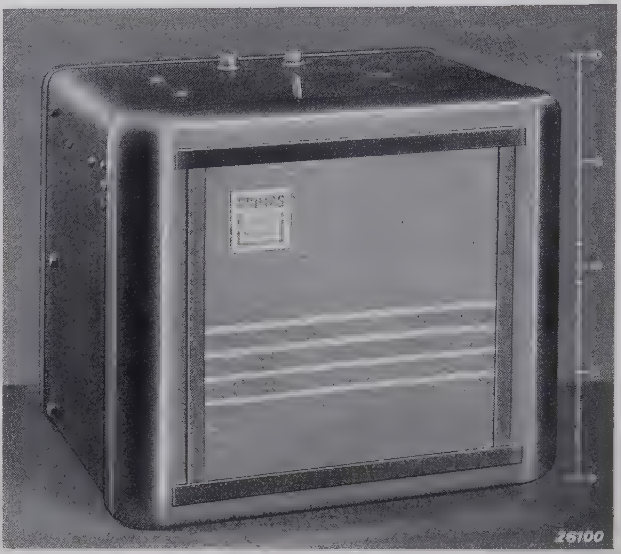

Fig. 4. A car radio set which is only 18,5 cm high.

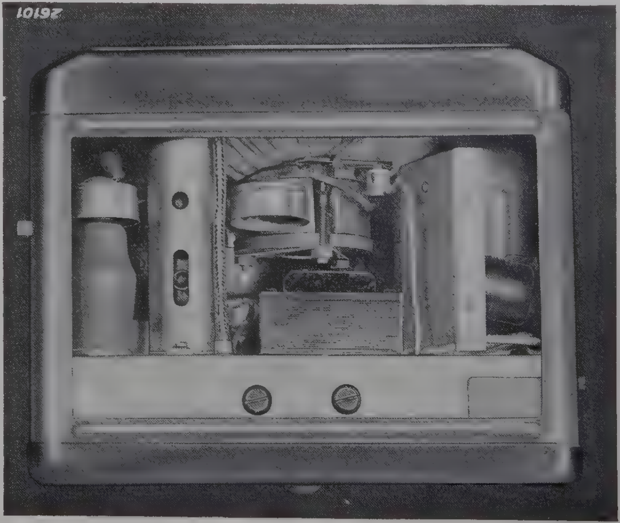

Fig. 5. View of the apparatus with a side wall cut open. Two valves have been removed from the middle in order to show that part of the apparatus which lies behind them.

box (fig. 7) is for tuning, and is connected by a Bowden cable with a worm in the set which turns the shaft of the tuning condenser. The transmission from the Bowden cable to the pointer is in the operating box itself. The dial is of a special type, since the requirement must be made that the tuning be easily visible at night as well as in the daytime, and moreover that it must not radiate light in such a way as to be disturbing. When driving in the dark an entirely lighted tuning scale is very disturbing; for that reason the following design has been chosen. The scale itself is dark with light letters (for reading in the daytime); the light letters allow light to pass through and are lighted from behind at night. Between the two wavebands there is a lighttransmitting band upon which a light coloured stripe is visible as a pointer in the daytime. This pointer is placed in the middle of a light transmitting sector of an otherwise non-transparent disc beneath the scale. Under this disc is the lamp for illumination. In this way only a sector-shaped part of the scale is lighted, and the correct tuning point is then indicated by the shadow of the pointer stripe on a light background. In addition, by using a light green lamp for the illumination, a pleasant, non-distracting but very clear reading of the scale is made possible.

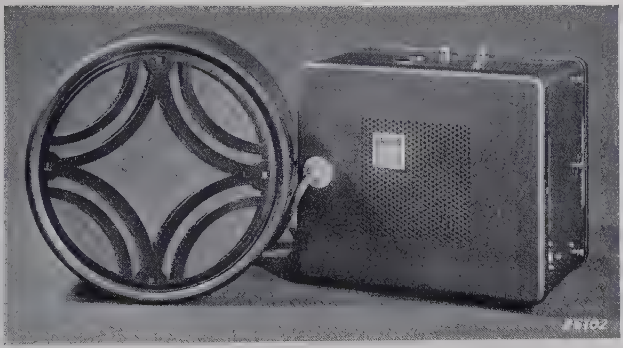

Fig. 6. Car radio set with separate loudspeaker.

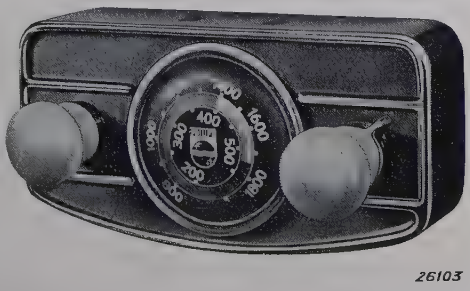

Fig. 7. Operating box.

On the operation box itself (fig. 7) to the right under the tuning knob there is also a switch by which the power supply to the apparatus can be switched on. The lefthand knob operates the volume control of the apparatus, also through a Bowden cable. For switching over to a different wave length range there is no extra knob. The volume regulator knob, when pulled out slightly, performs this function.

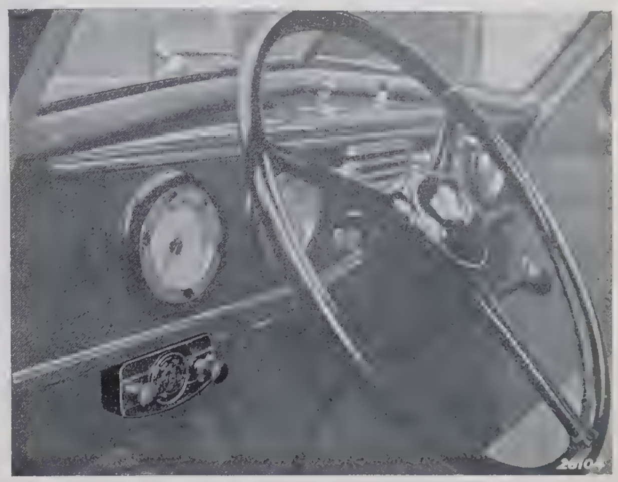

Fig. 8. Operation box under the dashboard.

3) The apparatus must be very sturdily built, since at high speeds and especially on bad roads, it may be exposed to very heavy shocks. Because of this the parts must be extra well fastened. During assembly this feature is checked by placing the sets on a vibrating table, each corner of which falls once per second a distance of 12 mm. The four corners do not fall at the same moment, but one falls every quarter of a second. The construction is

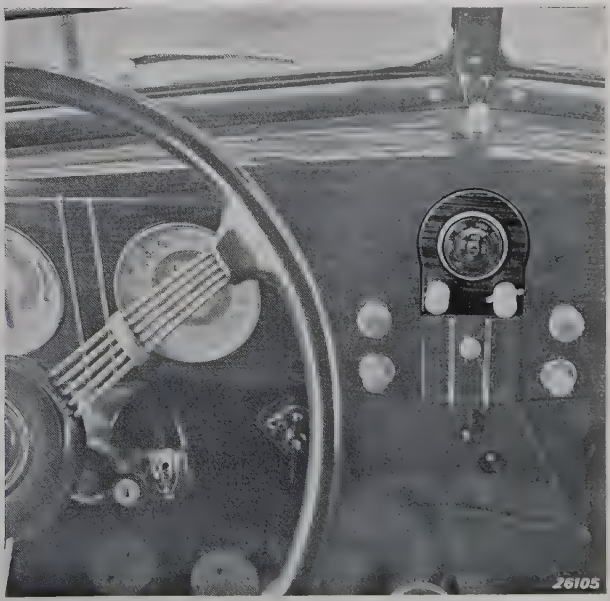

Fig. 9. Operation box in the dashboard.

such that the sets can withstand such shock tests for hours.

4) The apparatus must be easy to install. In order that this may be done quickly and without too many changes in the car, a single bolt method of installation has been chosen (*fig. 10.*) With this method it is only necessary to drill one hole in the partition behind the motor. The separate loudspeaker has the same type of fastening. In addition the cable connections for supply and aerial are made with a bayonet fastening which limits the time necessary for their assembly to a minimum.

5) The apparatus must be completely closed electrically. If the apparatus is suspended below the dashboard, it is close to connections which carry strong interference voltages. As has been mentioned in the discussion of interferences, the apparatus itself must be closed to such interferences. The housing is therefore of metal, and there is a wire gauze under the cloth screen of the loudspeaker. Since the front cover must be removable for the purpose of changing valves when necessary, there is an open crack between cover and housing. This is, however, entirely closed by spring strips. There are also such strips between the housing and the lower cover, since the latter must sometimes also be removed for inspection.

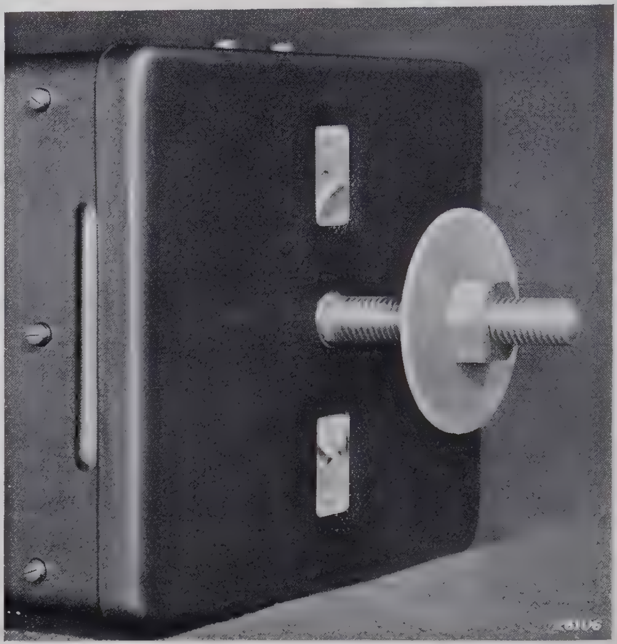

Fig. 10. The car radio set is fixed with a single bolt to the partition behind the motor.

THE SEALING OF METAL LEADS THROUGH HARD GLASS AND SILICA

by B. JONAS.

666.1.037.5 : 666.19

A survey is given of the saeling of metal leads through different kinds of glass. The various conditions necessary for obtaining a tight and heat-resisting lead-in through quartz glass fused silica are discussed.

Properties of various glasses

At the beginning of the present century, after it had become possible to manufacture clear transparent fused silica on a large scale, the material quickly became indispensable in chemical and physical laboratories. Its success is due to its remarkable resistance to sudden changes of temperature and to chemical attack, and to its unusually satisfactory optical and electrical properties even at high tempeatures.

Until recently, however, silica was not used on a large scale in the construction of electric lamps or discharge tubes. There is a tendency to ascribe this fact to the relatively high price or to the technical difficulties of its use; this is, however, incorrect. The cause may rather be seen in the fact that at that time there was no perfectly good method of introducing current leading-in wires through the silica.

In order to show where the difficulties of this problem lie we shall compare it with the analogous problem in the case of ordinary glass. For this purpose the factors which are important in the process of sealing in are collected in *Table 1* to gether with some other data. The numerous technically important kinds of glass are so arranged in the table that "quartz" appears as the last member of a series, and no emphasis is laid on a contrast between "ordinary glass" and fee sed silica. It is obvious that this classification is somewhat arbitrary, for instance, in the establishment of the boundaries between the different types; moreover, special cases are omitted for the sake of clearness. Some of

Table 1. PHYSICAL PROPERTIES OF SILICATE GLASSES

	Group No.	Class	Sum SiO_2 and Al_2O_3 by weight %	Region where softening begins: Transformation point °C	Softening point °C	Coefficient of linear thermal expansion $\alpha \times 10^7$	Tensile strength kg/mm²	Resistance to compression kg/mm²	Heat conductivity cal/cm degr.sec.	Electrical conductivity T_{k100} °C	Dielectric losses at 1.5×10^6 cycles/sec. $10^4 \cdot tg\delta$	Transmissibility for ultra-violet of 1 mm of glass at 3000 Å
Ordinary soft glass	1	Lead glass (alkali lead silicate glass)	35 to 65	380 to 500	420 to 600	85 to 95	3 to 7	50 to 90	0.017 to 0.037	250 to 380	8 to 80	0% to 75 %
	2	Calcium glass (alkali calcium silicate glass)	65 to 75	470 to 530	550 to 600	80 to 110	4 to 10 (15)	70 to 100	0.025 to 0.030	130 to 300	10 to 120	
Hard glass	3	Soft boro-silicate glass (alkali boro-silicate glass)	70 to 85	470 to 570	500 to 700	30 to 60				160 to 400		
	4	Hard alumino (boro) silicate glass poor in or free from alkali	75 to 95	650 to 900	750 to 1000	8 to 38	5 to 15	100	approx. 0.03	350 to 600	approx. 10 to 30	up to 90%
		Quartz glass	100	1050	1300	5.5 to 6	>20(?)	>200	0.035	>600	2	92%

the values given in the table, for instance the tensile strength, are somewhat doubtful for various reasons, but they serve to give a general idea.

When one speaks of a "hard glass" one is not referring to the mechanical hardness, but to the fact that the glass is difficult to fuse and must be worked at a high temperature. In recent years the the term "hard glass" has also been used to indicate the fact that the coefficient of thermal expansion is small. By the term "soft hard glass" we shall in the following indicate a glass which has a small coefficient of expansion, but which becomes soft at a relatively low temperature.

By "transformation point" we mean the lowest temperature at which, after lengthy heating, it is still possible to relieve the stresses in the glass; at the softening temperature this can be done in a few minutes (for example in 15 minutes).

Instead of the conductivity at room temperature, the temperature T_{k100} is given, at which a conductivity g of 1/100 mhos/cm is reached.

Different forms of the metal-to-glass joint

All glass-to-metal joints must satisfy the requirement that the stresses, which appear during manufacture or in normal use due to the difference in coefficients of expansion of the materials used, shall not lead to a loosening of the joint or to the destruction of one of the parts. Joints obtained by cementing, mechanical methods or by electrolytic methods are not considered here, because of their low resistance to heat. It may be assumed that with a really good prepared glass-to-metal joint the adhesive strength is not lower than the tensile strength of the glass. When this is true the problem of the glass-to-metal joint is reduced mainly to the limitation of the stresses appearing in the glass.

In the solution of this problem two fundamentally different lines may be chosen:

1. by judicious choice of the two components glass and metal, the coefficients of thermal expansion are so adapted to each other that the final state is stable. (This does not mean that the coefficients of expansion exactly must be equal).
2. by taking very small thicknesses of metal, dangerous stresses are avoided in the glass. In this method no regard is paid to exceeding the limit of elasticity of the metal.

The second method is of only slight importance in the scaling in of wires, since with an appreciable difference in expansion the permissible thickness of wire is too small. For the fusing on of ring- or cap-shaped parts this method is very often used, especially when there are not many or important temperature changes at the boundary of glass and metal. In the case of fused silica such a construction would be accompanied by very great difficulties, considering the extremely high temperature during working. In general this method must be considered more as a subsidiary to the more usual process in the case of a large diameter of the metal part.

A method which can also be followed is the employment of a flux which is adapted to the glass of the tube, and has such a low softening temperature that deviations in the coefficient of expansion are permissible. The temperature interval through which the weld still cools after becoming solid then becomes shorter, and the thermal stresses also become smaller. In the cases where this method is of practical importance it comes down to a facilitation of methods 1) or 2), but it is not in principle a different solution.

Finally, when it is found impossible to adapt the cofficients of thermal expansion of glass and metal to each other, it is possible to bridge the difference by a series of kinds of glass with progressive properties. This solution can be used with fused silica (*fig. 1a*) and is very important for laboratory purposes. It is, however, not cheap, and in the case of lamps it leads to rather elaborate constructions, so that it cannot be considered for mass production.

In the following we shall confine ourselves to the primary form of a weld: the leading in of a wire or rod through a glass wall, a type of construction which can be achieved with all kinds of glass, even with pure silica. We shall now discuss the

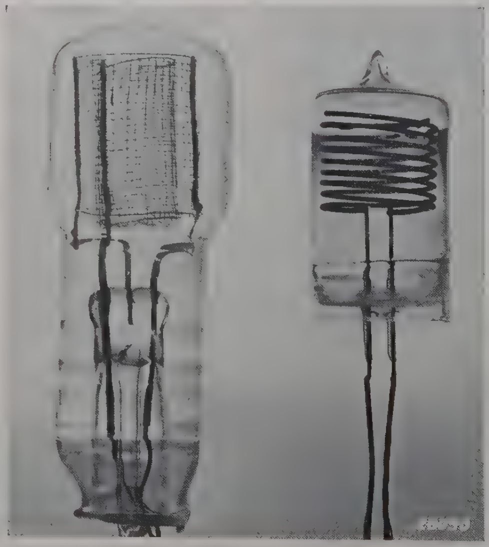

Fig. 1. Scaling of tungsten into silica bulbs with the aid of: *a*) transition by means of gradually changing kinds of glass, *b*) flux glass with a high melting point ($^3/_4$ of real siz

process with reference to Table I successively dealing with glasses of different degrees of hardness.

Sealing leads through "soft glasses" and "soft hard glasses"

In the case of the first two groups in the table I (soft glasses) limitation of the stress by adaptation of the thermal expansion gives no difficulties. For the region of coefficients of expansion from 80 to 110 × 10^{-7}, in addition to platinum and its alloys, there are several series of alloys with gradually varying properties. In addition there are composite metal wires, such as copper clad wire, which can also be adopted.

In order to prevent the leading-in wires, which unfortunately seldom consist of highly conducting metals, from becoming too hot, the limiting load is usually 10, or at the most 20 A per wire. If the load is not continuously and artificially cooled, upon exceeding this value there is danger of a break due to electrolysis of the glass (from about 200° C), since the conductivity of the glass increases rapidly with the temperature. In such cases a cap-shaped or cylindrical piece must be used, with highly conducting metals for the current leads.

For the third group in the table, the "soft hard glasses", the solution is also usually simple. With the softer kinds, whose thermal expansion is as a rule not particularly low, adaptable sealing-in alloys can again be used; with many of these glasses the metals tungsten and molybdenum can also be used. The appropriate alloys are ternary or quaternary Fe-Ni-Co alloys derived from the well-known Invar (approximately 64 Fe, 36 Ni). It is well known that Invar has a very small thermal expansion at room temperature. This is the result of a reversible thermal transformation which is complete at about 200°C. The expansion curve, which is practically flat to about 150°C, has the normal steeply mounting form above 200°C (*fig. 2*). By the addition of cobalt as the third constituent the transition from the flat to the steep part can be displaced to higher temperature, while the slope of the flat part increases. Such alloys can therefore — if necessary with the addition of other components such as Mn, Cu or Cr — be adapted for use with the softer kinds of glass of the third group. For the harder kinds of the third group the beginning of softening often lies at a much higher temperature than the bend in the expansion curve of the alloy. The disadvantage arising from this can be avoided by the introduction of a softer intermediate flux glass between the glass in question and the metal, when tungsten or molybdenum cannot be used.

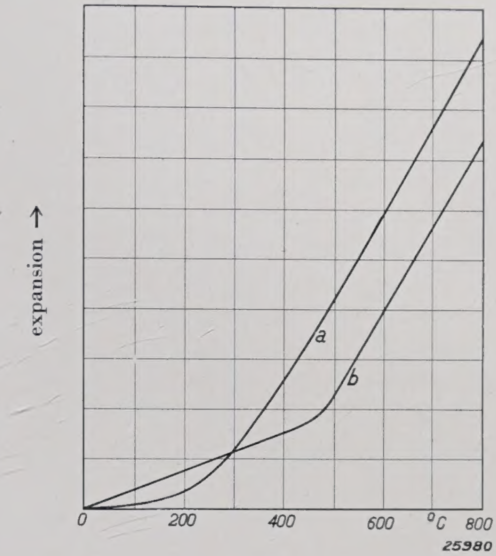

Fig. 2. Diagram of the variation of thermal expansion: *a*) Invar (36 Ni, 64 Fe), *b*) a Co-Ni-Fe alloy resembling Invar.

Summarizing, we may therefore say that for the three groups of softer glasses considered, a satisfactory solution can always be found no matter what the desired ratio of coefficients of thermal expansion may be (within certain limits). For the permissible value of this ratio, in the literature and in practice, a range is usually taken from 0.9 to 1.1. Sometimes one finds the permissible difference $\alpha_{\text{metal}} - \alpha_{\text{glass}}$ given in the literature as 20 × 10^{-7}.

Wire leads through hard glass and fused silica

In the case of the last two groups, if we wish to keep to the practical rule that the coefficients of expansion of the materials to be fused together, metal and glass, may not differ by more than 10 per cent, the problem of sealing in leads through these kinds of glass is insoluble. Of all the available metals, tungsten with 47 × 10^{-7}, has by far the lowest, and thus the most favourable, coefficient of expansion. Upon comparison with the kinds of glass now being considered, there are differences of 20 to 40 × 10^{-7}, and the ratio mentioned would therefore become 1.6 to 8 instead of 1.1. Moreover, the difference in coefficient of expansion is active over a much larger region, since the softening temperature is so much higher.

Upon closer consideration of the nature and magnitude of the stresses occurring, however, it is found that they do not exceed the limit of strength of the glass or of the glass-metal joint under circumstances which are not all too unsatisfactory.

For simple forms of the joint, as given in *fig. 3*, the stresses can be quantitatively determined by means of the double refraction of the glass under stress in polarized light. Such measurements have been carried out by Hull and Burger ¹) with joints of various dimensions and of many different kinds of glass and wire. The values measured were also compared with those calculated on the basis of simple assumptions, and were found to give a good agreement.

We shall now apply this method of calculation to the lead-in through hard glass of the fourth group which was not studied by Hull and Burger, as we believe that the method will also in this case give a good picture of the situation.

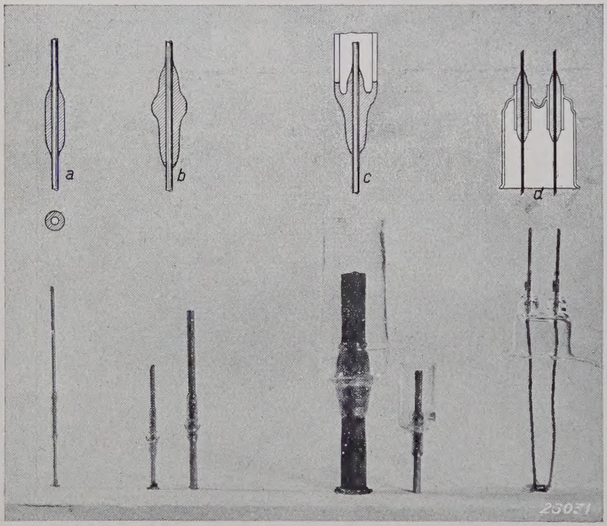

Fig. 3. Various forms of lead-in. Photographs and sectional drawings (scale 1 : 3).

Calculation of the distribution of stress

The calculations are based upon the following model: a long glass cylinder containing a concentric wire core. At the lower limit of the softening region of the glass the whole is in temperature equilibrium and without stress. We assume that upon further cooling only elastic changes in shape occur.

The metal core has the tendency to shrink more upon cooling than the hard glass covering. *Fig. 4* shows how in the glass tensile stresses occur in a radial direction and compressive stresses in tangential (and axial) directions. For a glass (see table I) the permissible loading on compression is about ten times that on stretching. In the model considered the ratio between the tensile and compressive stress is usually ²) less than 10, so that rupture will occur only due to tensile stresses, and we shall concern ourselves in the following only with the latter (in the radial direction).

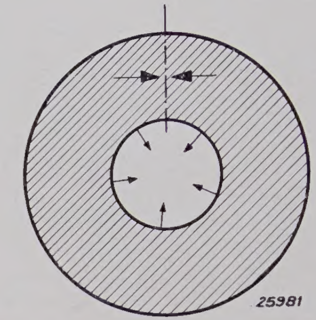

Fig. 4. Stresses occurring in glass part of lead-in, when $a_g < a_m$.

The radial stress σ_r as a function of the distance r to the axis is given by the equation:

$$\sigma_r = \frac{p}{n^2 - 1}\left(\frac{r_g{}^2}{r^2} - 1\right),$$

where r_g is the radius of the glass cylinder and n is the ratio of this radius to the radius of the metal core.

The radial stress σ_r is zero at the outer surface of the glass ($r = r_g$), and has a maximum p at the boundary glass-metal, where the ratio $r_g/r_m = n$. In passing it must be noted that in the metal core the radial stress is constant and equal to the maximum value p of the stress in the glass. In *fig. 5* the variation of σ_r/p is given as a function of the ratio r/r_g for several values of n.

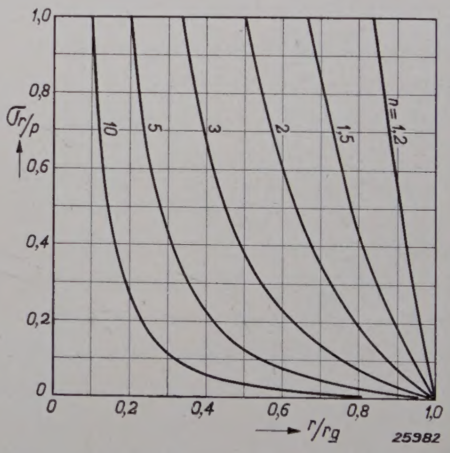

Fig. 5. Relative value of the tensile stress (σ_r/p) as a function of (r/r_g) for various values of n ($= r_g/r_m$).

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¹) A. W. Hull and E. E. Burger, Physics 5, 384 - 405, 1934.

²) With a very thin layer of glass the tangential (compressive) stress can become greater than 10 times the radial (tensile) stress. The value of the compressive stress then occurring is, however, according to calculation, always smaller than twice the maximum tensile stress with a very thick layer of glass. The axial stresses are never greater than the tangential ones at the boundary metal-glass. It is therefore sufficient to restrict the tensile stress.

The function which indicates the variation in the stress is the same as that which appears in the well-known case of a thick-walled tube with internal excess-pressure. The determination of the maximum stress p, however, demands a deeper study of the particular circumstances of the sealing-in problem. This has been discussed by Poritsky [3]) whose results were used by Hull and Burger after several simplifications and a correction.

From the calculation it follows that the maximum stress p is proportional to the product of the modulus of elasticity of the glass E_g and the difference δ in expansion per unit of length between glass and metal for the temperature interval under consideration. Further p depends on the ratio n $(= r_g/r_m)$ and on the elastic properties of metal and glass.

$$p = \frac{E_g \cdot \delta}{\varrho_g \left(\frac{E_g}{E_m}\right) + \frac{\gamma n^2 + \varrho_m}{n^2 - 1}}$$

E_g, E_m are respectively the moduli of elasticity of glass and metal, while γ and ϱ are connected with Poissons's ratia μ of glass and metal.

$$\gamma = \frac{(1+\mu_g)\ [1 + (n^2-1)\, E_g/E_m]}{(1+\mu_g) + (n^2-1)\,(1+\mu_m)\, E_g/E_m} \approx 1$$

$$\varrho_g = 1 - 2\mu_g;\ \varrho_m = 1 - 2\mu_m$$

The order of magnitude of ϱ_g and ϱ_m is 0.5.

It may perhaps seem strange that the stress p on the inner side of the glass does not depend explicitly on the dimensions of the wire fused in or of the glass surrounding it, but only on the ratio n. Experience of course shows that it is more difficult to make a "heavy" lead-in than a "light" one. This fact is, however, a result of the increasing difficulties in the manipulation and cooling, when the dimensions of the piece of work become larger.

The expression given for p enables us to calculate the dangerous stresses in a weld with an accuracy depending upon the reliability of the elastic constants and the assumptions concerning n. As an example we have chosen one of the most unfavourable cases out of group 4, an alumino-boro-silicate glass which, as experience has shown, can easily be sealed to quartz. With $E_g = 6700$ kg/mm², $E_m = 36000$ kg/mm², $\delta = 3.0 \times 10^{-3}$, $\mu_g = 0.25$ and $\mu_m = 0.17$, we find for different values of n the numbers given in *table 2* (cf. *fig.* 6). As was to be expected, the values found for p are very high, for a high value of n the breaking limit of this glass appears to be attained or exceeded. In practice, however, the joints under consideration hold well, even at a high temperature and upon sudden large changes in temperature. We shall investigate the reason for this unexpectedly high resistance.

Table 2.

Maximum stress p (kg/mm²) occurring with a simple leading-in wire through an alumino-borosilicate glass (group 4) with $E_g = 6700$ kg/mm², $E_m = 3600$ kg/mm², $\delta = 3 \times 10^{-3}$, $\varrho_g = 0.5$, $\varrho_m = 0.66$.

n —	1.0	1.2	1.5	2.0	3.0	5	10	∞
p —	0	3.5	6.9	10.1	12.4	12.9	14.5	15.5

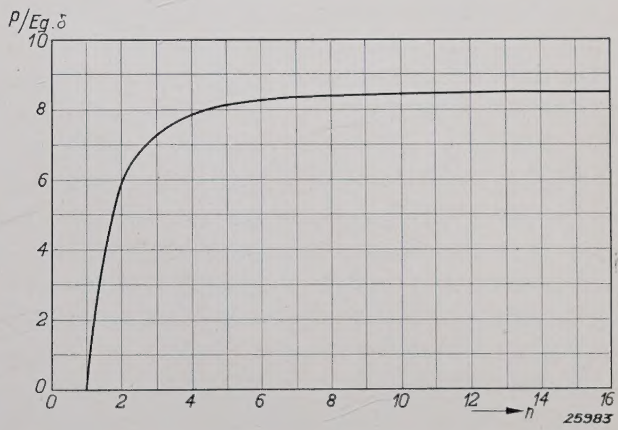

Fig. 6. Ratio $p/E\delta$ as a function of n $(= r_g/r_m)$. $E_g/E_m = 0.186$, $\varrho_g = 0.5$, $\varrho_m = 0.66$.

In the first place it may be noted that we began the calculation with the assumption of a state without stress at a definite (high) uniform temperature. This assumption of a uniform temperature does not agree with the actual fact. Due to the high temperature during working and to the good heat conduction of the metal (tungsten), a temperature gradient occurs in a radial direction toward the axis. The metal then shrinks less later on, and the maximum stress p is reduced, so that the values calculated are to be considered as an upper limit which is never actually reached.

The incidental factors do not always act in a favourable sense, that is, to decrease the maximum stress. Due to uneven cooling the stresses occurring may become greater than the value calculated. This occurs when $\alpha_m > \alpha_g$, and the glass covering cools more rapidly than the metal wire. In the case which we are considering, on the other hand, a fall of temperature toward the axis is ensured during working.

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[3]) H. Poritsky, Physics 5, 406 - 411, 1934.
G. Heller has called our attention to several errors in the derivation given in the article referred to. In the formulae used here the necessary corrections have been made.

A possible doubt as to the strength of the adhesion between glass and metal calls for care. When $a_m > a_g$ the maximum tensile stress occurs at the boundary surface, and even a local loosening of the seal between metal and glass can easily lead to rupture of the glass.

The sealing of tungsten to the kinds of glass here discussed gives a weld which is one of the strongest known with respect to the adhesion of metal to glass. If, however, no details are known about the adhesion, this factor must not be assumed to be too high, and a smaller difference in coefficients of expansion must be chosen.

Under certain circumstances much higher tensile stresses may be encountered in glass than the maximum values determined in the usual way from tensile tests. In such tests a great variation in the results is found which is usually ascribed to uncontrollable injuries and faults in the glass surface. With glass rods whose surface is already under tangential and axial compression before being loaded, due to a sudden cooling, appreciably higher values of the tensile strength and resistance to bending occur, even though there are previous tensile stresses in the core.

An analogous phenomenon is known in the case of the so-called "Prince Rupert's Drops" which can be made by allowing drops of glass to fall into a cooling liquid. The surface of the drops is under a high compressive stress. From optical measurements it may be deduced that extraordinarily high tensile stresses occur in the interior of the drops. This same heat treatment is applied technically to the "hardened" safety glass. In these examples, due to the avoidance of tensile stresses at the surface, an abnormally high tensile strength occurs at the core. With seals of tungsten through glass with a lower coefficient of expansion an analogous distribution of stresses is obtained, at least when the surface of contact between metal and glass need not be considered as a glass surface. As long as glass and metal adhere, all practical experience seems to support this conclusion. Because of these favourable circumstances a much higher tensile stress for the glass may be assumed than is usual.

It has actually been found possible to make reliable leads of tungsten through almost all the kinds of glass in the fourth group, even with the glass referred to in Table 2 with excellent results. It thereby becomes possible to introduce lead-in wires into silica walls by means of a weld which is tight and resistant to heat, since some of the glasses in group 4 can be sealed to silica. With pure silica alone, it is, according to calculation, which is confirmed by experience, impossible to make a tight weld, at least of ordinary dimensions.

Fig. 3 shows the practical form of a lead-in of tungsten through fused silica with the aid of an intermediate glass with a high melting point (see also fig. 1b). It may be seen that remarkably "heavy" welds can be made. In this connection we finally call attention to the very high current load permissible. If the temperature of the wire increases by development of heat in the wire or by conduction of heat from the inside of the tube, the stresses in the glass become smaller, in contrast to the usual case of equal expansion ($a_m = a_g$).

Thanks to this decrease in stress it is possible to seal in leads for currents up to many hundred amperes. The current is limited by the oxidation of tungsten occurring at dark red heat, and to a smaller degree by the danger of electrolysis, which with the glass in question, only becomes of importance at still higher temperatures.
